Suppose that we want to estimate the consumption function

$$c_t^* = \beta_1 + \beta_2 y_t^*$$

where c_t^* and y_t^* are true but unobservable consumption and income, respectively. The observable values c_t and y_t are subject to errors of measurement and are related to the true values as follows:

$$c_t = c_t^* + v_t \qquad y_t = y_t^* + u_t$$

where v_t and u_t are each independent and identically distributed random variables with $N(0, \sigma_v^2)$ and $N(0, \sigma_u^2)$ distributions, respectively. Data on c_t, y_t, and two potential instrumental variables, i_t (investment) and g_t (government expenditure), appear in Table 14.3. ("DATA" ON WEBSITE)

(a) Find least squares estimates of β_1 and β_2. (OLS)

(b) Find instrumental variables estimates for β_1 and β_2 by using the following instruments:

(i) i_t (ii) g_t (iii) $x_t = i_t + g_t$ (iv) i_t and g_t

Comment on the alternative estimates and their standard errors.

(c) A number of variations of Hausman's specification test for testing for contemporaneous correlation between y_t and the composite error $v_t - \beta_2 u_t$ are possible. These variations depend on *(i)* the instrumental variable (IV) estimator that is used, and *(ii)* whether the error variance $\text{var}(v_t - \beta_2 u_t) = \sigma^2$ is estimated from least squares procedures or instrumental variables procedures. Using estimates for β_2, carry out Hausman's test for the following cases. Comment on the outcome.

Case	IV Estimator	Choice of $\hat{\sigma}^2$
1	Uses i_t	IV
2	Uses g_t	IV
3	Uses x_t	Least squares
4	Uses x_t	IV
5	Uses i_t and g_t	IV

Table 14.3 Hypothetical Data for i, g, c, and y

Observation	i	g	c	y
1	1.5	0.5	15.30	17.30
2	1.4	0.6	19.91	21.91
3	1.5	0.7	20.94	22.96
4	1.4	0.8	19.66	21.86
5	1.5	0.9	21.32	23.72
6	1.4	1.0	18.33	20.73
7	1.6	1.0	19.59	22.19
8	1.5	1.1	21.30	23.90
9	1.6	1.2	20.93	23.73
10	1.6	1.2	21.64	24.44
11	1.7	1.3	21.90	24.90
12	1.6	1.4	20.50	23.50
13	1.8	1.4	22.83	26.05
14	1.7	1.5	23.49	26.69
15	1.9	1.5	24.20	27.60
16	1.8	1.6	23.05	26.45
17	2.0	1.6	24.01	27.61
18	1.9	1.7	25.83	29.43
19	2.0	1.8	25.15	28.95
20	2.0	1.8	25.06	28.86