Suppose, we have $Y_1, Y_2, \dots Y_T$ i.i.d. random variables (r.v) with mean β and variance σ_y^2.

THEORY:

mean of Y_t : $E(Y_t) = \beta$

variance of Y_t : $E(Y_t - E(Y_i))^2 = \sigma_y^2$

PRACTICE = ESTIMATION:

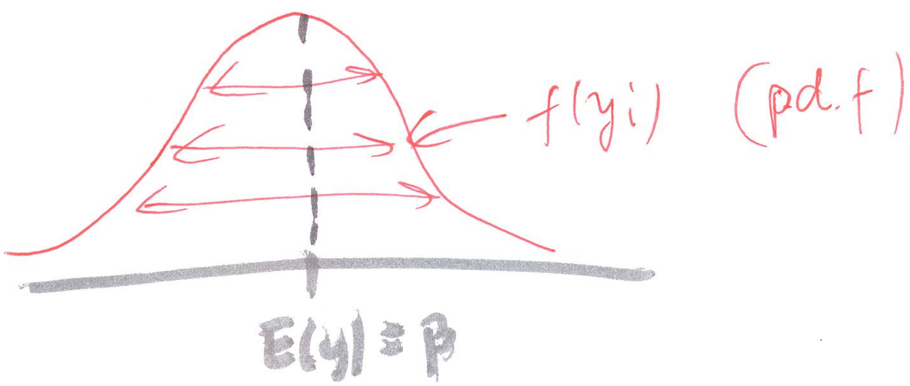
Suppose we have a random sample of observations of Y_i: $y_1, y_2, \dots y_T$

- mean estimator

$$\bar{Y} = \hat{\beta} = \frac{1}{T}\sum_{t=1}^{T} y_t$$

- variance estimator

$$\hat{\sigma}_y^2 = \frac{1}{T}\sum_{t=1}^{T}\left(y_t - \hat{\beta}\right)^2$$



get the data: $y_1, y_2 \cdots y_T$

② use OLS to estimate β

$$y_t = \beta + e_t \quad , \quad e_t \sim (0, \sigma_e^2)$$

OLS: $$\min \sum_{t=1}^{T} e_t^2 = \boxed{\min \sum_{t=1}^{T} (y_t - \beta)^2}$$

$$\min S(\beta) = \sum_{t=1}^{T} (y_t - \beta)^2$$

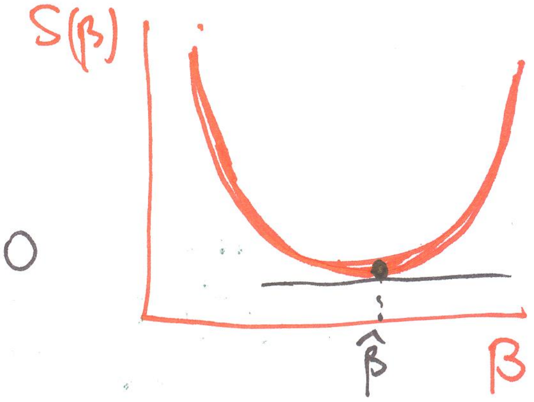
$$= \sum_{t=1}^{T} y_t^2 - 2\beta \sum_{t=1}^{T} y_t + T\beta^2$$

$(a-b)^2$

$$\sum_{t=1}^{T} \beta^2 = T \cdot \beta^2$$

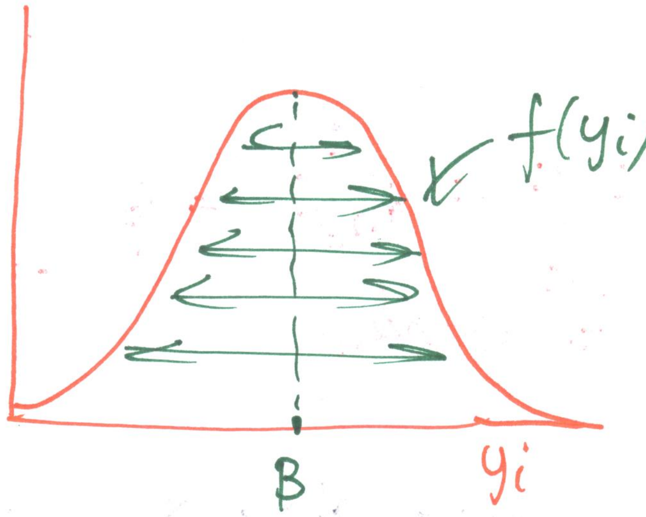
$$\frac{\partial S(\beta)}{\partial \beta} = -2 \sum_{t=1}^{T} y_t + 2T\beta$$

$$\frac{\partial S(\beta)}{\partial \beta} = 0 \Leftrightarrow -2 \sum y_t + 2T\beta = 0$$



replace β by $\hat{\beta}$, b?

$$\boxed{\hat{\beta} = \frac{\sum y_t}{T} = \frac{1}{T} \sum y_t = \bar{y}}$$



$$\beta \downarrow \hat{\beta} = \bar{y}$$

$$\sigma_y^2 \rightarrow \hat{\sigma}_y^2$$

Properties of the estimator:

• $E(\hat{\beta})$ we want $E(\hat{\beta}) = \beta$

if yes, $\hat{\beta}$ is UNBIASED

$$E(\hat{\beta}) = E\left(\frac{\sum y_t}{T}\right) = \frac{1}{T}\sum_{t=1}^{T} E y_t = \frac{1}{\cancel{T}}\cancel{T}\beta = \beta$$

$$\circ \ var(\hat{\beta}) = var\left(\frac{1}{T}\sum y_t\right) = \left(\frac{1}{T}\right)^2 var \sum y_t$$

$$= \frac{1}{T^2}\sum_{t=1}^{T} var\, y_t$$

$$var(\hat{\beta}) = \frac{1}{T^2}\, T \cdot \sigma_y^2 = \frac{\sigma_y^2}{T}$$

efficient estimator = minimum variance estimator.

OLS $\rightarrow$ yes!

in this model only! $\sigma_e^2 = \sigma_y^2$. It estimates the MARGINAL mean and variance of Y_t.

An alternative model for y_t:

Suppose: y_t : spending

x_t : income

$$y_t = \beta_1 + \beta_2 \underline{\underline{x_t}} + e_t$$

ASSUMPTIONS ON e_t:

$$e_t \sim (0, \sigma_e^2)$$

$$E(e_t \mid x_t) = 0$$

allows us to estimate the CONDITIONAL = MODEL BASED MOMENTS OF y_t:

$$E(y_t \mid x_t) = \beta_1 + \beta_2 x_t)$$

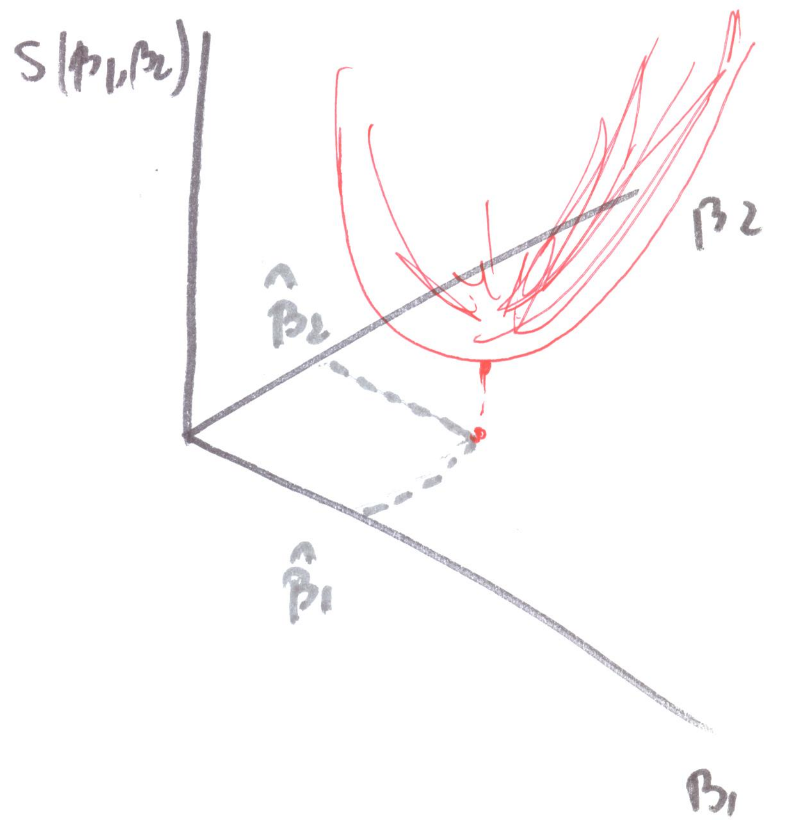
$$\mathrm{var}(y_t \mid x_t) = E\left(y_t - \underline{(\beta_1 + \beta_2 x_t)}\right)^2 = E(e_t^2) = \sigma_e^2$$

$$= E\left[\left(y_t - \underline{E(y_t \mid x_t)}\right) \mid x_t\right]^2$$

$$y_t = \beta_1 + \beta_2 x_t + e_t, \quad e_t \sim (0, \sigma_e^2)$$

OLS:

$$\min S(\beta_1, \beta_2) = \min \sum_{t=1}^{T} e_t^2 = \min \sum_{t=1}^{T} (y_t - \beta_1 - \beta_2 x_t)^2$$



$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$$

$$\hat{\beta}_2 = \frac{\widehat{\text{cov}}(y_t, x_t)}{\widehat{\text{var}}\, x_t} = \frac{\cancel{\frac{1}{T}} \sum (y_t - \bar{y})(x_t - \bar{x})}{\cancel{\frac{1}{T}} \sum (x_t - \bar{x})^2}$$

properties of OLS

Greene.

$$E(\hat{\beta}_1) = \beta_1 \qquad E(\hat{\beta}_1 \mid x_t)$$

$$E(\hat{\beta}_2) = \beta_2 \qquad E(\hat{\beta}_2 \mid x_t)$$

unbiased

$$y_t = \beta + e_t$$

$$y_t = \beta_1 + \beta_2 x_t + e_t$$