ANSWER 3 OF THE FOLLOWING 4 QUESTIONS

QUESTION 1

Consider the seemingly unrelated regression (SUR) model:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

where $e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \sim N(0, W)$, and $W = \begin{bmatrix} \sigma_1^2 I_T & \sigma_{12} I_T \\ \sigma_{12} I_T & \sigma_2^2 I_T \end{bmatrix}$. The $T \times K$ matrix X_1 and $T \times K$ matrix X_2 contain regressors fixed in repeated samples.

(a) Show that when $\sigma_{12} = 0$, the variance of the GLS estimator of the entire model is equal to the variances of OLS estimators obtained separately from each equation of the system.

(b) How would you test the null hypothesis $H_0 : \sigma_1^2 \neq \sigma_2^2$?

(c) Suppose that the null hypothesis that the coefficients on variables in equation 1 are equal to the coefficients on the equivalent variables in equation 2 has not been rejected by the test.

Given that outcome, a "pooled regression" OLS estimator of the common coefficient vector β is proposed as an alternative to the GLS estimator in a). It is obtained by regressing

$Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ on $X^* = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

Compare the properties of this OLS estimator and the FGLS estimator of the model: 1) are they unbiased/consistent and 2) what can be said about the variances of those estimators.

QUESTION 2

An econometrician needs to estimate a linear model with the dependent variable y_t and explanatory variable z_t^*:

$$y_t = \beta_1 + \beta_2 z_t^*$$

which is referred to as Model (1). The random variable z_t^* is not directly observed. The data available for this variable contain a measurement error:

$$z_t = z_t^* + u_t$$

where the measurement error u_t has mean zero and variance σ_u^2, and is serially uncorrelated and independent of z_t^* and y_t.

a) Specify Model 1 in terms of the observable variables and define the error term e_t. You have obtained Model 2.

   all questions below concern Model 2

b) Write the fomulas of the OLS estimator b_2 and its variance.

c) Is the OLS estimator b_2 a consistent estimator of β_2? Show and explain what affects the consistency of that estimator.

d) The econometrician wants to use variable x_t as an instrument for z_t and to proceed in two steps: First regress z_t on x_t, and next use the fitted value $\hat{z}_t$ in the regression of y_t on $\hat{z}_t$. If the correlation between x_t and z_t is close to 0, what implication(s) would that have on the IV estimator $\hat{\beta}_2^{IV}$?

QUESTION 3

Consider the simultaneous equation model:

$$\begin{aligned} y_{1t} &= \beta_{11}x_{1t} + \gamma_{12}y_{2t} + e_{1t} && (1) \\ y_{2t} &= \beta_{21}x_{1t} + \gamma_{21}y_{1t} + \beta_{22}x_{2t} + \beta_{23}x_{3t} + e_{2t} && (2) \end{aligned}$$

where errors e_{1t} and e_{2t} have variances σ_1^2 and σ_2^2, respectively, and are uncorrelated.

(a) Show that coefficient γ_{12} cannot be consistently estimated by the OLS from equation (1).

(b) Use the identification conditions to determine which equation is estimable

(c) Write the reduced form (without deriving it -use the symbols π_{ij} for the coefficients of the reduced form) and describe the 2SLS estimation.

(d) Suppose that variables x_2 and x_3 are very highly correlated. How is that problem called in econometrics and how would it affect the identification and the 2SLS estimation-discuss.

QUESTION 4

Consider the following regression model with a single random regressor:

$$y_t = \beta_1 + \beta_2 x_t + e_t,\ t = 1, ..., T \tag{3}$$

where the errors e_t are iid, with mean zero, and variance σ^2. The regressor x_t may be correlated with the error term e_t.
Denote by X the $T \times 2$ matrix that contains a vector of ones and the vector x_t.

1) If $plim\frac{1}{T}X'e = 0$, explain why the econometrician should use the OLS estimator b_{OLS} rather than the IV estimator $\hat{\beta}_{IV}$.

2) Suppose, that an econometrician decides to test the null hypothesis H_0 : $plim\frac{1}{T}X'e = 0$, using the Hausman test based on the difference of estimators, denoted by $d = \hat{\beta}_{IV} - b_{OLS}$. Under H_0, $plim\ d = 0$, whereas under the alternative $plim\ d \neq 0$. The test statistic is

$$m = d'\{\widehat{Var}(d)\}^{-1}d.$$

Show that $Var(d) = Var(\beta_{IV}) - Var(b_{OLS})$, using the following result by Hausman: "under the null hypothesis, *the covariance between an efficient* *estimator b_E of a parameter vector β, and its difference $b_E - b_I$ from an inefficient estimator b_I of the same parameter vector is zero".*

3) Generalize your result in to the estimated variances, and show that

$$m = \frac{1}{\hat{\sigma}^2} d'[(\hat{X}'\hat{X})^{-1} - (X'X)^{-1}]^{-1} d$$

4). Under H_0 we are using two different, but consistent estimators of σ^2. Why should we use the same error variance estimator, such as $\hat{\sigma}^2_{OLS}$ in the estimated variances of estimators.