Fig 1.2 Monthly CRSP Share Price 1925/12 to 1989/12

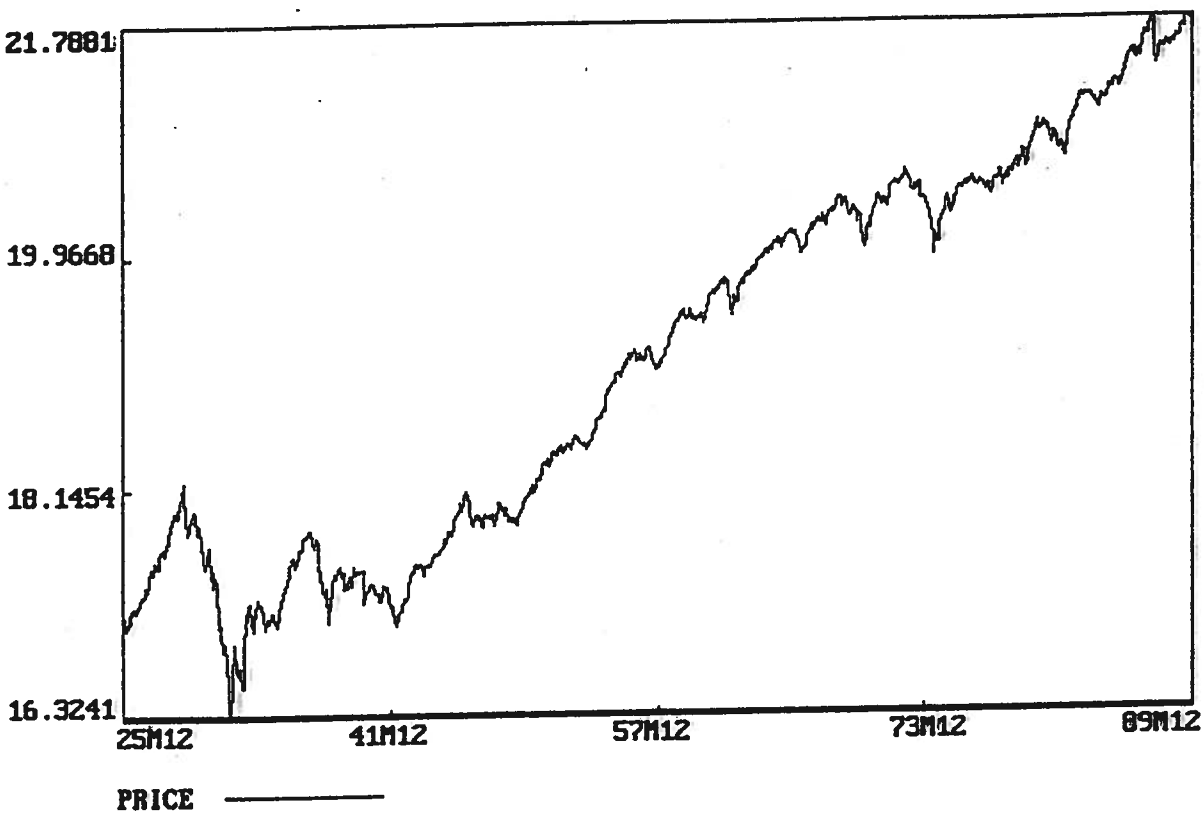


Fig 1.5 Monthly CRSP Equity Returns 1926/1 to 1989/12

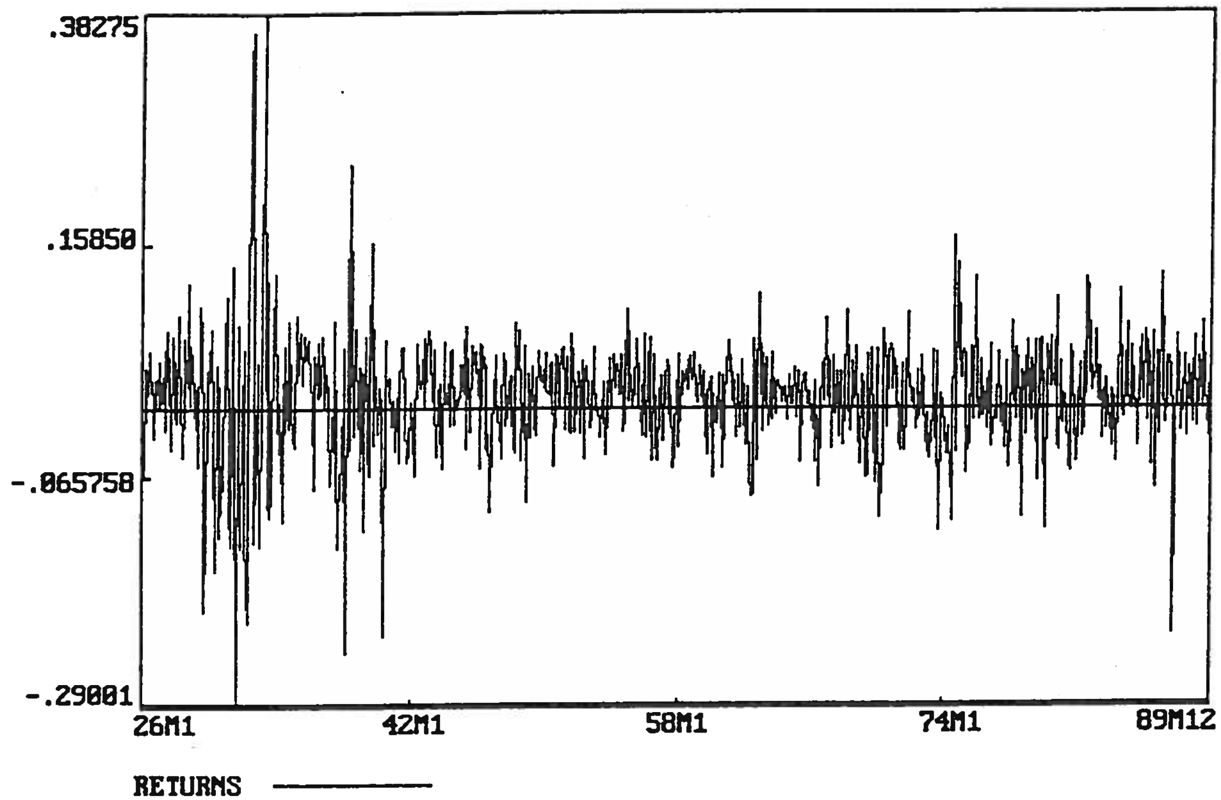


Fig 1.1 Time Series of Gaussian Noise

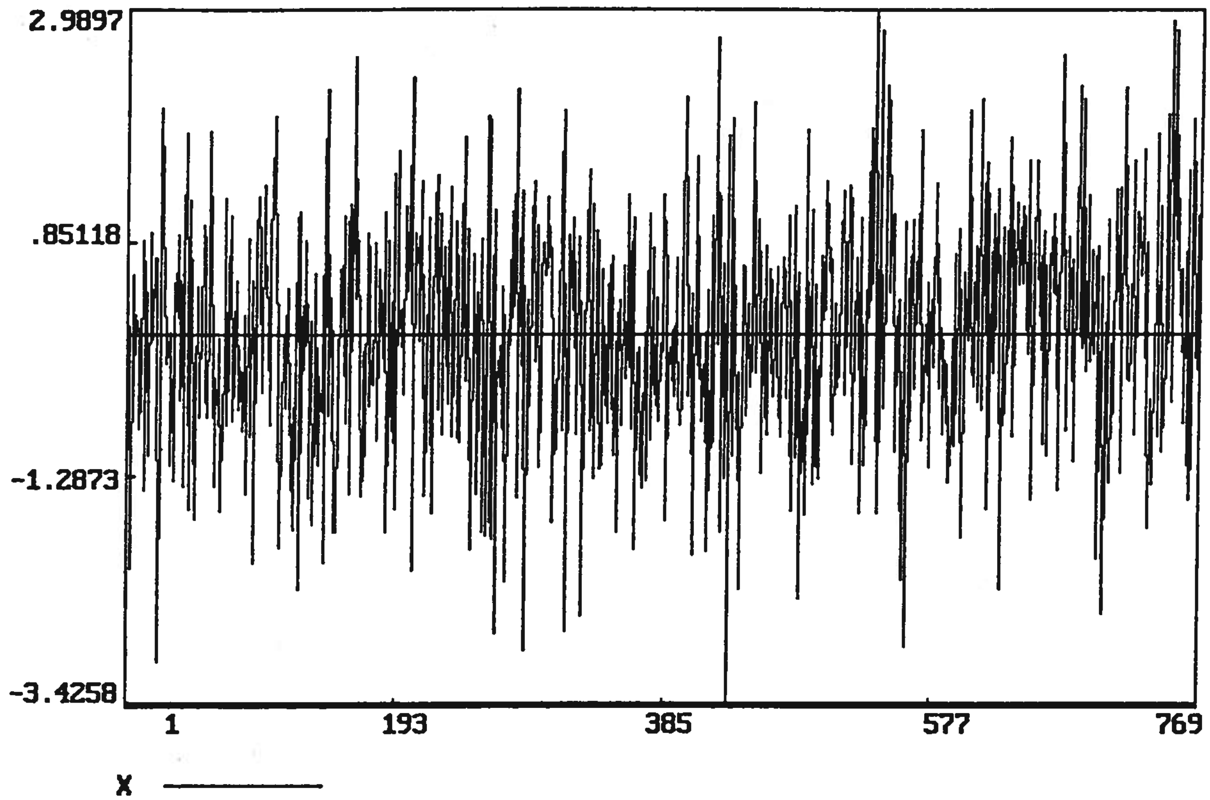
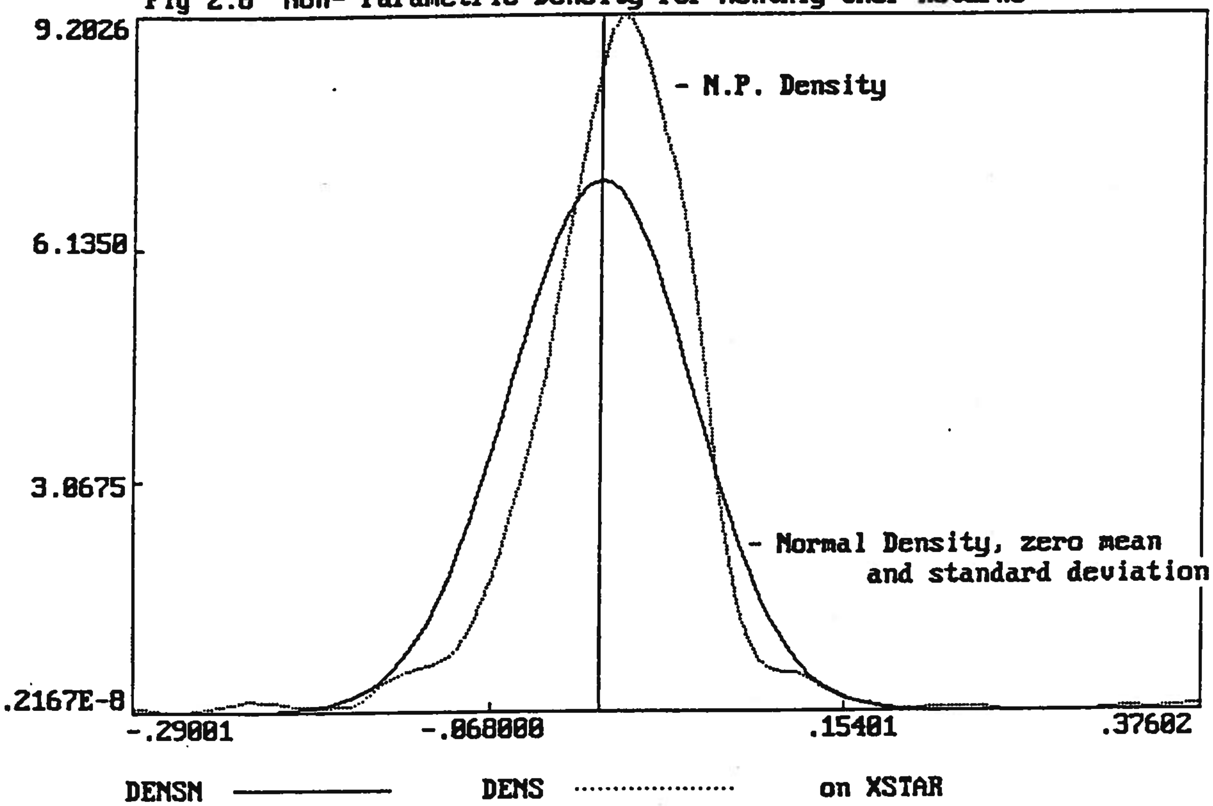


Fig 2.8 Non- Parametric Density for Monthly CRSP Returns



1.2 Empirical Regularities of Asset Returns

Parametric specification of $\varepsilon_t(\theta_0)$ has many possibilities

$\Rightarrow$ to have any guidance, study stylized facts first

(i) Thick tails

Asset returns are leptokurtic

(ii) Volatility Clustering

$\Rightarrow$ look at any plot of financial time series, i.e. stocks, exchange rate, etc.

French, Schwert and Stambaugh (1987) Volatility during 30's
> Volatility during 60's

Note : Thick tails and volatility clustering are related

(iii) Leverage effects

Black (1976)

Leverage effect = tendency for changes in stock prices to be negatively correlated with changes in stock volatility.

Firm with debt and equity becomes highly leveraged when value of the firm falls.

(iv) Non-Trading Periods

Fama (1965), French and Roll (1986).

Information accumulates more slowly when markets are closed.

(v) Forecastable Events

Forecastable events of important info $\Rightarrow$ high ex ante volatility.

Indiv. firm's stock returns volatility around earnings announcements.

Also fixed income and foreign exchange volatility increase during periods of central bank trading.

Volatility at the open and close of market and middle of day.

(vi) Volatility and Serial Correlation

Strong inverse relation between volatility and serial correlation for U.S. stock indices.

(vii) Co-movements in volatilities

Commonality in volatility.

$\Rightarrow$ reason for factor ARCH

(viii) Macroeconomic Variables and Volatility

Surprisingly weak link between macroeconomic incertainty and volatility.

Glosten, Jagannathan and Runkle (1993).

Strong positive relationship between stock return volatility and interest rates.

ESTIMATION : ARCH(1)

$$y_t = \beta_0 + e_t$$

$$e_t \mid \text{past} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2$$

$$\Rightarrow \frac{e_t}{\sqrt{h_t}} \sim N(0,1)$$

let us denote $\frac{e_t}{\sqrt{h_t}}$ by z_t

density of e_t:

$$f(e_t) = f(z_t) \cdot \left| \frac{\partial z_t}{\partial e_t} \right| = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{e_t^2}{2h_t}\right) \cdot \left| \frac{1}{\sqrt{h_t}} \right|$$

log-likelihood function:

$$\ln L(e_1 \dots e_T) = -\frac{T}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=2}^{T} \ln h_t - \frac{1}{2} \sum_{t=2}^{T} \frac{e_t^2}{h_t}$$

replace unobservable e_t by $y_t - \beta_0$

$$\ln L(y_1 \dots y_T) = -\frac{T}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=2}^{T} \ln\left(\alpha_0 + \alpha_1 (y_t - \beta_0)^2\right) - \frac{1}{2} \sum_{t=2}^{T} \frac{(y_t - \beta_0)^2}{\alpha_0 + \alpha_1 (y_t - \beta_0)^2}$$

can be maximized w.r.t $\beta_0, \alpha_0, \alpha_1$

let $\theta = \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \alpha_1 \end{bmatrix}$

Maximization of MLE: moving from θ^0 to θ^1 in each iteration.

is based on the Taylor expansion of L at θ^0:

$$L(\theta^1) = L(\theta^0) + \underbrace{\left[\frac{\partial L(\theta)}{\partial \theta}\right]_{\theta_0}}_{\text{gradient: } D(\theta^0)} (\theta^1 - \theta_0)$$

$$L(\theta^1) = L(\theta^0) + D'(\theta^0)(\theta^1 - \theta^0)$$

set $\underbrace{\theta^1 - \theta^0}_{}$ $= \underbrace{\lambda}_{\text{"step"}} \underbrace{[M(\theta^0)]}_{\text{hessian}}^{-1} D(\theta^0)$

Hessian: $\left[\frac{\partial^2 L}{\partial \theta \partial \theta'}\right]_{\theta_0}$. Negative def so change sign.

$$\theta^1 = \theta^0 - \lambda \left[M(\theta^0)\right]^{-1} D(\theta^0)$$

algorithm stops when gradient $= 0$ i.e. $D(\theta^0) = 0$