step 1: test the residuals to check for cointegration:

$$\hat{y}_t = \beta x_t + \hat{e}_t$$

$$\hat{e}_t$$

$$\Delta \hat{e}_t = \underline{\underline{\gamma}}\, \hat{e}_{t-1} + v_t$$

τ ratio: and look in the cointegration table ($\underline{\underline{-2.76}}$)

$\hat{e}_t$ are stationary (reject unit root")

① y_t, x_t are cointegrated

② regression $y_t = \beta x_t + e_t$ IS VALID AND REPRESENTS THE LONG RUN EQUILIBRIUM stationarity :

eigenvalues of $\Phi = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}$ are less

than 1 in modulus.

$$Z_{t-1} = Z_t \cdot L$$

$$Z_t = \Phi Z_{t-1} + V_t$$

$$Z_t - \Phi Z_{t-1} = V_t$$

$$Z_t \left(Id - \Phi L \right) = V_t$$

Stationarity condition: determinant of the matrix $(Id - \Phi L)$ is polynomial in lag operator L of 2nd order:

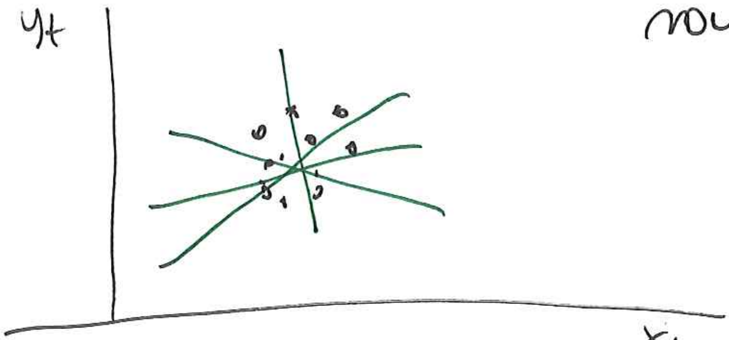
$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \beta_{11}L & \beta_{12}L \\ \beta_{21}L & \beta_{22}L \end{bmatrix} \right)$$

$$= \begin{pmatrix} 1-\beta_{11}L & -\beta_{12}L \\ -\beta_{21} & 1-\beta_{22}L \end{pmatrix}$$

the determinant of this matrix has roots outside the unit circle.

no linear relationship: = "long run equilibrium"

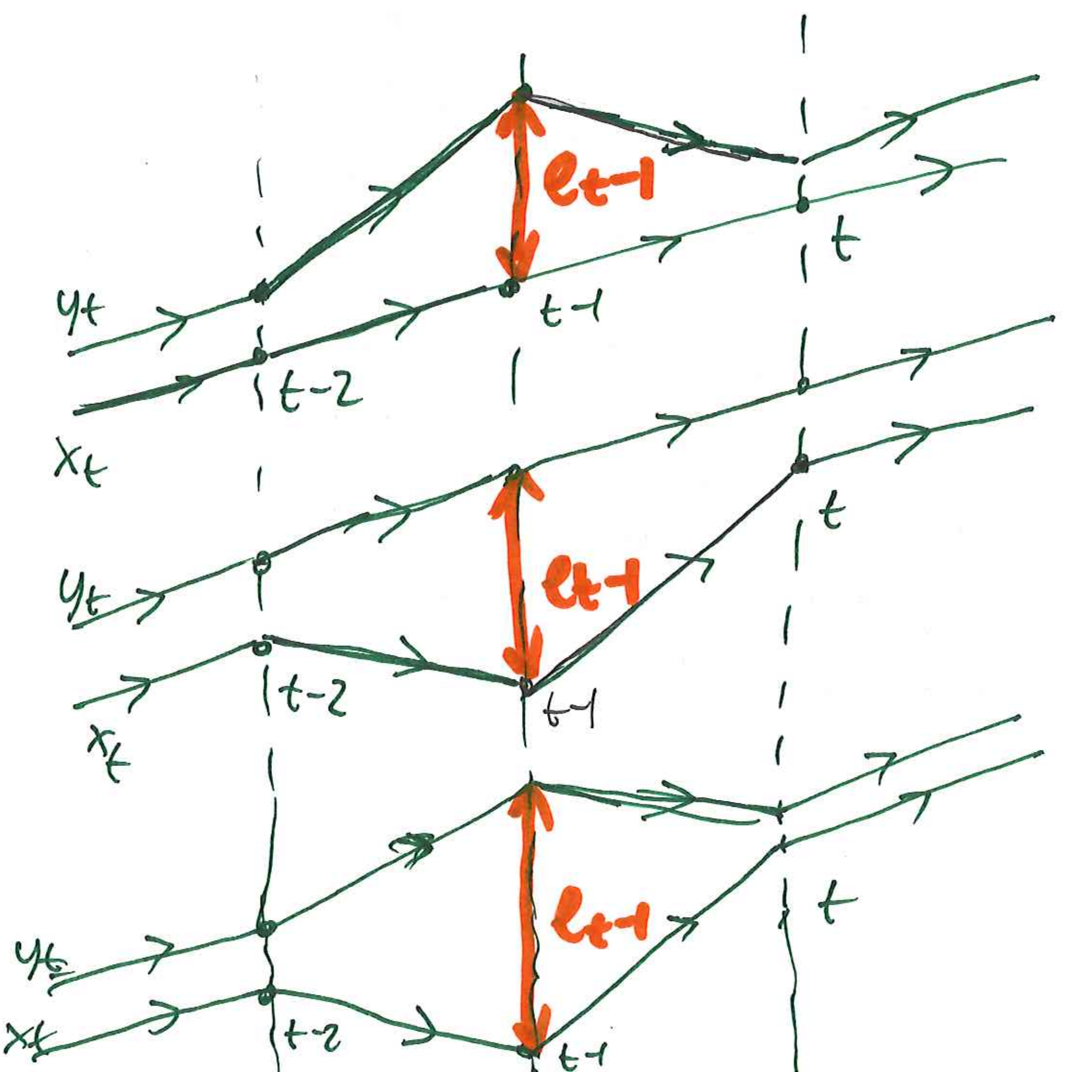
round scatterplot



- how does the "long run equilibrium" remain satisfied over time?
- what makes it hold and last at each t?

ERROR CORRECTION MECHANISM (ECM/

(VECTOR) ERROR CORRECTION (VEC)

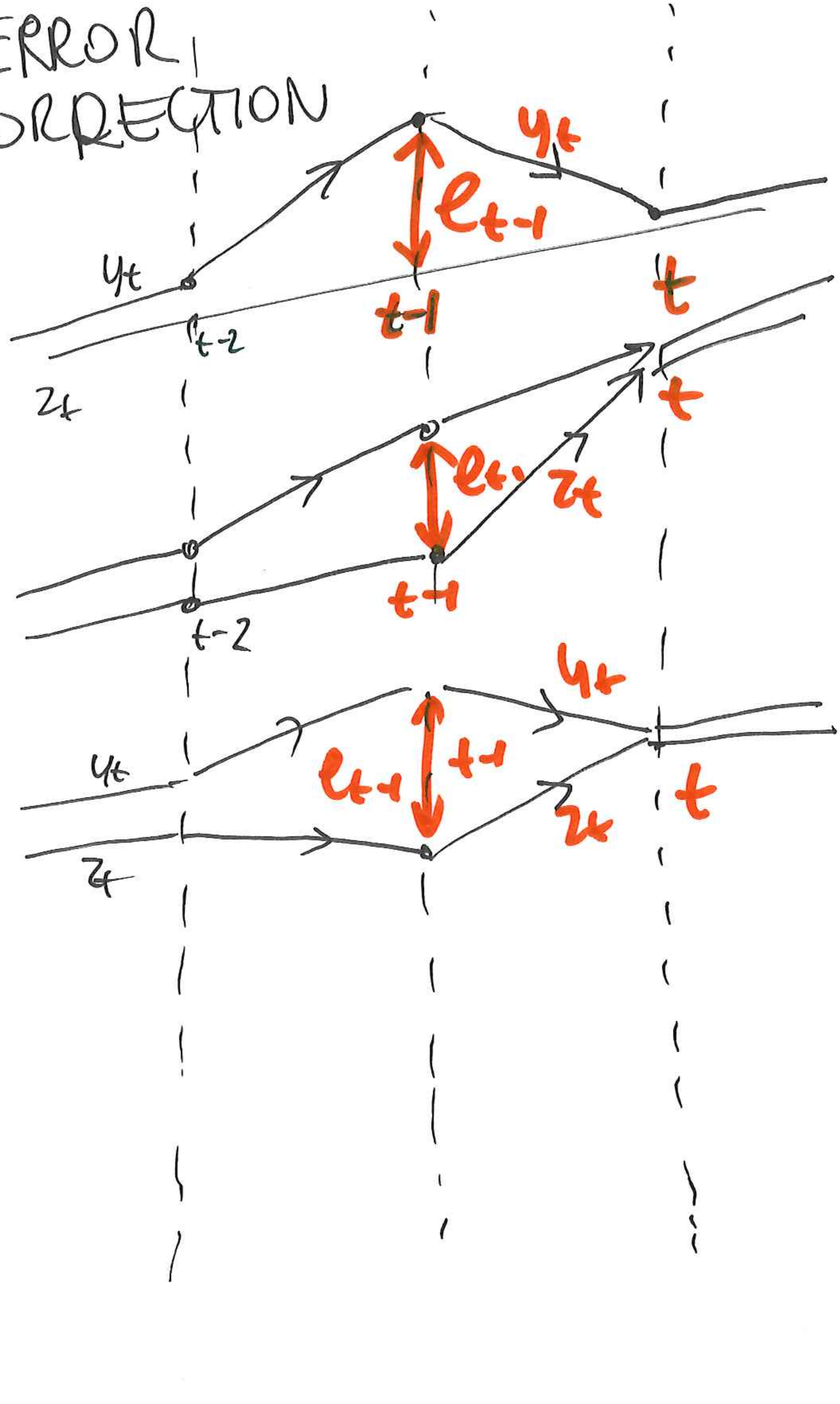


y_t does the correction only

x_t does the job

x_t, y_t both close the gap: e_{t-1}

ERROR CORRECTION



$$y_t = \beta_0 + \beta x_t + e_t$$

$$\hat{e}_{t-1} = y_{t-1} - \hat{\beta}_0 - \hat{\beta} x_{t-1}$$

$$\Delta y_t = \alpha_{10} + \alpha_{11} e_{t-1} + v_t^y$$

$$\Delta x_t = \alpha_{20} + \alpha_{21} e_{t-1} + v_t^x$$

$$\Delta y_t = \alpha_{10} + \alpha_{11} y_{t-1} - \alpha_{11} \beta_0 - \alpha_{11} \beta_1 x_{t-1} + v_t^y$$

$$\Delta x_t = \alpha_{20} - \alpha_{21} y_{t-1} + \alpha_{21} \beta_0 + \alpha_{21} \beta_1 x_{t-1} + v_t^x$$

Granger Causality

does past x determine present y?

does past y determine present x?

→ can only be answered from estimation

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} x_{t-1} + v_t^y$$

$$x_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} x_{t-1} + v_t^x$$

H_0: "$X \not\Rightarrow Y$"   x does NOT cause y

H_0: "$\beta_{12} = 0$"

OR:

H_0: "$Y \not\Rightarrow X$"   Y does NOT cause X

H_0: "$\beta_{21} = 0$"

$$y_t = 0.7\, y_{t-1} + 0.2\, x_{t-1} + v_t^y$$

$$x_t = 0.2\, y_{t-1} + 0.7\, x_{t-1} + v_t^x$$

impulse responses.

suppose at time 1

$y_1 = v_1^y = \sigma_y = 1$ one-time shock to y_t of size 1

$x_1 = v_1^x = 0$

$t=1$ $y_1 = 1$

$x_1 = 0$

$t=2$ $y_2 = 0.7 y_1 + 0.2 x_1 = 0.7 \cdot 1 + 0.2 \cdot 0 = 0.7$

$x_t = 0.2 \cdot y_1 + 0.7 \cdot x_1 = 0.2 \cdot 1 + 0.7 \cdot 0 = 0.2$

$t=3$ $y_3 = 0.7 y_2 + 0.2 x_2 = 0.7 \cdot 0.7 + 0.2 \cdot 0.2 = 0.53$

$x_3 = 0.2 y_2 + 0.7 x_2 = 0.2 \cdot 0.7 + 0.7 \cdot 0.2 = 0.28$

etc.

Shock to variable x_t of size 1:

$y_1 = v_1^y = 0$

$x_1 = v_1^x = \sigma_x = 1$

<u>t=1</u>

$y_1 = 0$

$x_1 = 1$

<u>t=2</u>

$y_2 = 0.7 y_1 + 0.2 x_1 = 0.2$

$x_2 = 0.2 y_1 + 0.7 x_1 = 0.7$

<u>t=3</u>

$y_3 = 0.7 \cdot 0.2 + 0.2 \cdot 0.7 = 0.28$

$x_3 = 0.2 \cdot 0.2 + 0.7 \cdot 0.7 = 0.53$

Granger causality in ECM (2)

$$\Delta y_t = \alpha_{10} + \underline{\underline{\alpha_{11}}}\, e_{t-1} + \beta_{11}\,\Delta y_{t-1} + \underline{\beta_{12}\,\Delta x_{t-1}} + v_t^{y}$$

$$\Delta x_t = \alpha_{20} - \alpha_{21}\, e_{t-1} + \beta_{21}\,\Delta y_{t-1} + \beta_{22}\,\Delta x_{t-1} + v_t^{x}$$

H^0 $x \not\Rightarrow y$ "X NOT CAUSES Y"

H_0: $\alpha_{11} = \beta_{12} = 0$

H_0^0: $y \not\Rightarrow x$ "Y NOT CAUSES X

H_0: $\alpha_{21} = \beta_{21} = 0$