ECM

Estimation & Tests

$$\Delta x_t = \Pi x_{t-1} + \sum_{i=1}^{p-1} \Pi_i \Delta x_{t-i} + \varepsilon_t$$

$E\,\varepsilon_t = 0$

$E(\varepsilon_t \varepsilon_J') = \Sigma$ for $t = J$, 0 otherwise.

$$L(\Sigma, \Pi, \Pi_1, \dots \Pi_{p-1}) =$$

$$\left(-\frac{Tn}{2}\right)\log 2\pi - \left(\frac{T}{2}\right)\log|\Sigma|$$

$$-\frac{1}{2}\sum_{t=1}^{T}\left[\Delta x_t - \Pi x_{t-1} - \sum_{i=1}^{p-1}\Pi_i \Delta x_{t-i}\right]' \Sigma^{-1}$$

$$\cdot\left[\Delta x_t - \Pi x_{t-1} - \sum_{i=1}^{p}\Pi_i \Delta x_{t-i}\right]$$

- to test for lag length p: estimate the unrestricted model $\to \Sigma_U$ and restricted model $\to \Sigma_R$, calculate

$$(T-c)\left(\log|\Sigma_R| - \log|\Sigma_U|\right)$$

- under H_0 it is χ^2 distributed with deg of freedom = # of restrictions
- c = max # of regressors in the longest equation.
- do not use joint F tests for coefficients in Π and the Π_i's

Engle - Granger Approach

consider y_t, z_t

step 1: pretest each variable for the order of integration (same # of unit roots is required)

step 2: estimate the long-run relationship

$$y_t = \beta_0 + \beta_1 z_t + e_t$$

Keep the residual $\hat{e}_t$

$\hat{e}_t$: estimated deviation from equilibrium at t

If $\hat{e}_t$ is stationary then y_t, z_t are cointegrated.

run Dickey - Fuller test of a_1 in the regression

$$\Delta \hat{e}_t = a_1 \hat{e}_{t-1} + \varepsilon_t$$

no need for intercept as $\Sigma \hat{e}_t = 0$

- If accept $H_0: a_1 = 0 \Rightarrow$ conclude residuals have a unit root, no cointegration

- If reject $H_0: a_1 = 0 \Rightarrow \hat{e}_t$ stationary

$\Rightarrow$ COINTEGRATION $C(1,1)$ between y & z

USE TABLE "C" Residual Based Cointeg. Test"

Step 3 estimate ECM:

$$\Delta y_t = \alpha_1 + \alpha_y \big[\underbrace{y_{t-1} - \hat{\beta}_1 z_{t-1}}_{= \hat{e}_{t-1}} \big] + \sum_{i=1} \alpha_{11}(i) \Delta y_{t-i} + \sum_{i=1} \alpha_{12}(i) \Delta z_{t-i} + \varepsilon_{yt}$$

$$\Delta z_t = \alpha_2 + \alpha_z \big[\underbrace{y_{t-1} - \hat{\beta}_1 z_{t-1}}_{= \hat{e}_{t-1}} \big] + \sum_{i=1} \alpha_{21}(i) \Delta y_{t-i} + \sum_{i=1} \alpha_{22}(i) \Delta z_{t-i} + \varepsilon_{zt}$$

is a VAR in 1st differences.

- OLS can be applied equation by equation (since each equation contains the same regressors)

- all terms are stationary $\Rightarrow$
  t tests for α_y, α_z
  F tests for α_{jk}
  are valid.

step 4 check if $\hat{\varepsilon}_y$, $\hat{\varepsilon}_z$ are White Noise

- if there remains serial correlation
  increase the # of lags

eventually you end up having a Near-Var
then use GLS by the SUR argument (MLE)

- Speed of adjustment coefficients

  - if $\alpha_z = 0$ and if all $\alpha_{21}(i) = 0$

    $\Rightarrow \Delta y_t$ does not Granger cause Δz_t

  - If one of the adjustment parameters $= 0$, say $\alpha_y = 0$ then y doesn't respond to deviation from LT equilibrium and z_t does all the job: y_t is said to be weakly exogeneous

Granger Causality: y_t does not Granger cause z_t if lagged Δy_{t-i} do not enter the Δz_t equation and z_t does not respond to deviations from equilibrium (is weakly exogeneous)

- can use impulse response, variance decomposition
- Warning: results depend on the specification
  of the LT equilibrium + multistep procedure (