EXTENSIONS OF D-F TESTS.

1. TO TEST FOR UNIT ROOTS IN THE PRESENCE OF

   a) SERIALLY CORRELATED ERORRS

   b) heteroskedasticity

   ⇒ Phillips - Perron tesst.

2. To test for multiple unit roots:

   if you test for one root, while in reality there are 2 or more, the empirical size of test > nominal. that is not 5% but 20%.

   therefore less chance to find all roots if you first test for one, then for 2....

   Dickey - Pantula: sequential test that preserves the nominal size.

   (sequence of t-test has more power than a joint F test)

   1. H_0 : m roots against H_A : $(m-1)$ roots

      if rejected, then

   2. H_0 : $(m-1)$ roots against $(m-2)$

      etc.

STRUCTURAL CHANGE
(REGIME SWITCH)

low power of unit root test: cannot distinguish between a unit root and a regime switch.

true: $y_t = 0.5\, y_{t-1} + \varepsilon_t + D_L$

$$D_L = \begin{cases} 0 & \text{for } t = 1 \ldots 50 \\ 3 & \text{for } t = 51 \ldots 100 \end{cases}$$

fitted: $y_t = a_0 + a_1 y_{t-1} + e_t$

$\hat{a}_1$ is biased to 1 !

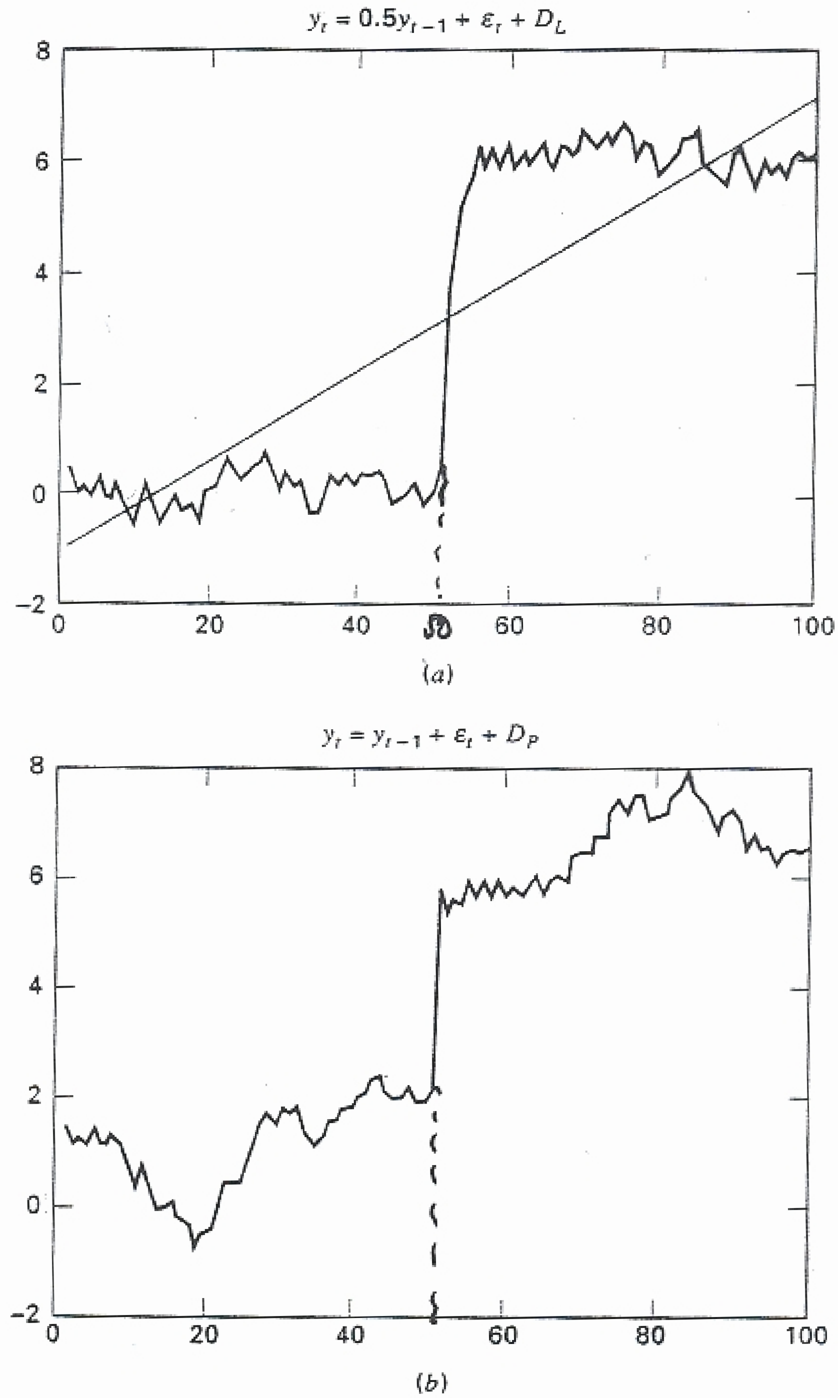
test is biased towards accepting H_0: unit root even though the series is stationary.

the fitted model mimics the best OLS regression

$$y_t = a_0 + a_2 t + e_t$$

$$\Leftrightarrow \quad y_t = y_0 + a_0 t + \sum_{i=1}^{t} \varepsilon_i$$

Figure 4.4 Two models of structural change.



tures the property that "low" values of y_t (i.e., those fluctuating around zero) are followed by other low values and "high" values (i.e., those fluctuating around a mean of 6) are followed by other high values. For a formal demonstration, note that as a_1 approaches unity, (4.30) approaches a random walk plus drift. We know that the solution to the random walk plus drift model includes a deterministic trend, that is, Random walk with a structural break:

$$y_t = y_{t-1} + \varepsilon_t + D_p$$

$D_p(51) = 4$ and 0 otherwise

Test procedure.

H_0: one-time jump in the level of a unit root process

H_A: a one-time change in the intercept of a trend stationary process.

$$H_0: y_t = y_0 + y_{t-1} + \mu_1 D_p + \varepsilon_t$$

$$H_A: y_t = a_0 + a_2 t + \mu_2 D_L + \varepsilon_t$$

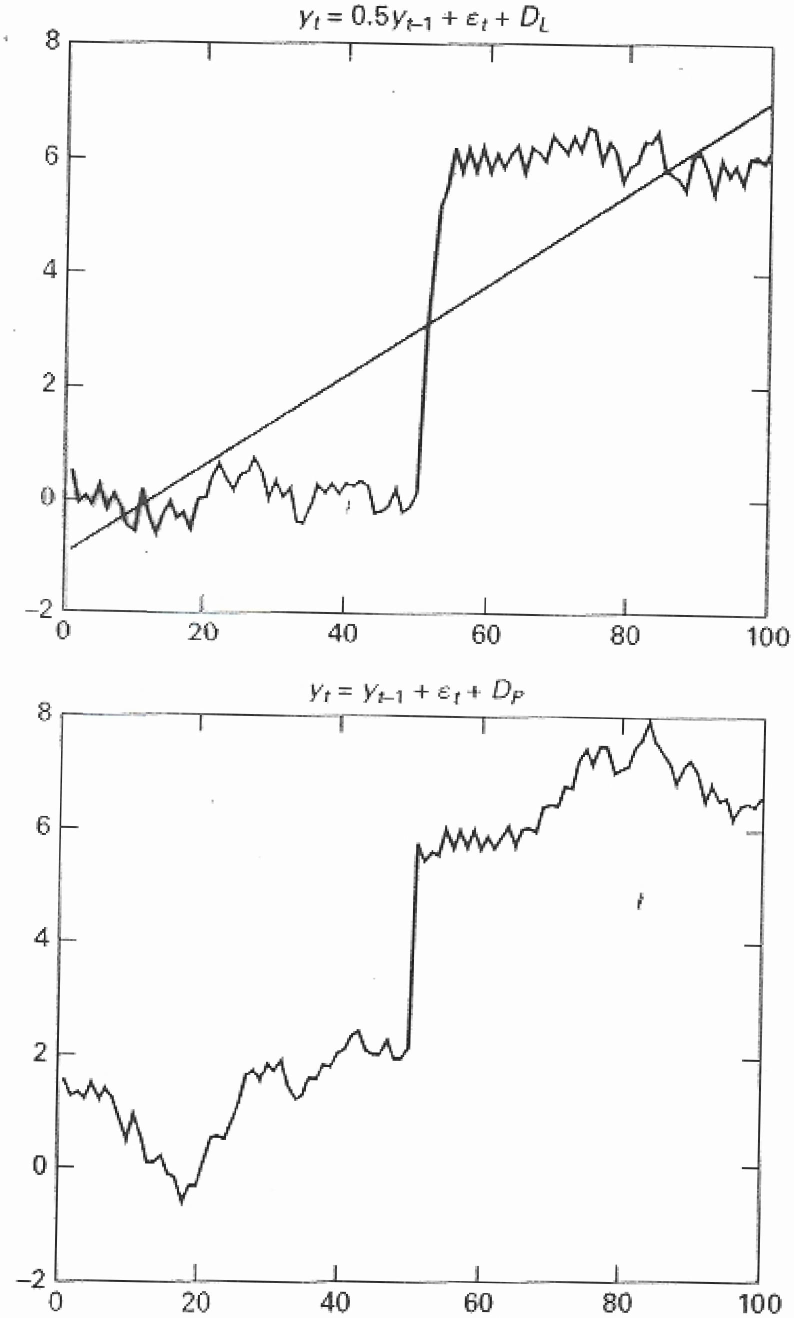


FIGURE 4.10 Two Models of Structural Change

$D_L = 1$ if $t > \tau$
0 otherwise

$D_P = 1$ if $t = \tau + 1$
0 otherwise

$H_0: y_t = a_0 + y_{t-1} + \mu_1 D_p + \varepsilon_t$

$H_A: y_t = a_0 + a_2 t + \mu_2 D_L + \varepsilon_t$

STEP 1: Detrend the data by estimating the model under the alternative hypothesis and calling the residuals $\hat{y}_t$. Hence, each value of $\hat{y}_t$ is the residual from the regression $y_t = a_0 + a_2 t + \mu_2 D_L + \hat{y}_t$.

STEP 2: Estimate the regression:

$$\hat{y}_t = a_1 \hat{y}_{t-1} + \varepsilon_t$$

Under the null hypothesis of a unit root, the theoretical value of a_1 is unity. Perron (1989) shows that when the residuals are identically and independently distributed, the distribution of a_1 depends on the proportion of observations occurring prior to the break. Denote this proportion by $\lambda = \tau/T$ where T = total number of observations.

STEP 3: Perform diagnostic checks to determine if the residuals from Step 2 are serially uncorrelated. If there is serial correlation, use the augmented form of the regression:

$$\hat{y}_t = a_1 \hat{y}_{t-1} + \sum_{i=1}^{k} \beta_i \Delta \hat{y}_{t-i} + \varepsilon_t$$

STEP 4: Calculate the t-statistic for the null hypothesis $a_1 = 1$. This statistic can be compared to the critical values calculated by Perron. Perron generated 5,000 series according to H_1 using values of λ ranging from 0 to 1 by increments of 0.1. For each value of λ, he estimated the regressions $\hat{y}_t = a_1 \hat{y}_{t-1} + \varepsilon_t$ and calculated the sample distribution of a_1. Naturally the critical values are identical to the Dickey–Fuller statistics when $\lambda = 0$ and $\lambda = 1$; in effect,

there is no structural change unless $0 < \lambda < 1$. The maximum differen between the two statistics occurs when $\lambda = 0.5$. For $\lambda = 0.5$, the critic value of the t-statistic at the 5 percent level of significance is -3.76 (whic is larger in absolute than the corresponding Dickey–Fuller statistic -3.41). If you find a t-statistic greater than the critical value calculated b Perron, it is possible to reject the null hypothesis of a unit root.

SPURIOUS REGRESSION

Granger, Newbold (1979)

$$y_t = a_0 + a_1 z_t + e_t$$

- has high R^2
- t statistics are significant
- no economic meaning

⇒ looks good because OLS INCONSISTENT
ALL INFERENCE IS WRONG.

- IF y_t, z_t have stochastic trends, $\{e_t\}$ may be nonstationary:

1) IF $\{e_t\}$ has a stochastic trend - each deviation is permanent since the error in any period t never decays.

$$y_t = y_{t-1} + \varepsilon_{yt}$$

$$z_t = z_{t-1} + \varepsilon_{zt}$$

$$e_t = y_t - a_1 z_t$$

$$e_t = \sum_{i=1}^{t} \varepsilon_{yi} - a_1 \sum_{i=1}^{t} \varepsilon_{zi}$$

- the $\text{var}(e_t) \to \infty$ as $t \to \infty$
- $E_t\, e_{t+1} = e_t$
- strongly correlated

A. IF $\{y_t\}$ $\{z_t\}$ INTEGRATED OF DIFFERENT ORDERS $\Rightarrow$ regression is meaningless.

B. $\{y_t\}$ $\{z_t\}$ INTEGRATED OF THE SAME ORDER AND RESIDUALS HAVE A STOCHASTIC TREND

$\Rightarrow$ estimate $\Delta y_t = a_1 \Delta z_t + \Delta e_t$

c. $\{y_t\}\ \{z_t\}$ INTEGRATED OF THE SAME ORDER AND $\{e_t\}$ STATIONARY $\Rightarrow$ COINTEGRATION

$$y_t = \mu_t + \varepsilon_{yt}$$

$$z_t = \mu_t + \varepsilon_{zt}$$

$$\mu_t = \mu_{t-1} + \varepsilon_t$$

$y_t - z_t = \varepsilon_{yt} - \varepsilon_{zt}$ IS STATIONARY