On-Line Appendices to
Generalized Covariance-Based Inference for Models Set-Identified from Independence Restrictions

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August 30, 2024
revised version

This document contains On-Line Appendices 1 and 2 with Closed-Form Expressions of Power Covariances and Additional Assumptions

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The authors gratefully acknowledge financial support of the Natural Sciences and Engineering Council of Canada (NSERC)

On-Line Appendix 1

Closed-Form Expressions of Power Covariances

Let us consider the framework of ICA with dimension 2 and the observations given by:

$$Y = Q'u,$$

where the rotation matrix is parametrized by the angle θ:

$$Q' = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

Then, we have

$$Y_1 = u_1 \cos\theta - u_2 \sin\theta, \;\; Y_2 = u_1 \sin\theta + u_2 \cos\theta.$$

We assume that the sources u_1, u_2 are independent, with means zero $Eu_1 = Eu_2 = 0$ and unit variances: $E(u_1^2) = E(u_2^2) = 1$. Then it is possible to derive the closed-form expressions of the power covariances up to power 3. We have:

$$\begin{aligned}
Cov(Y_1, Y_2) &= 0, \\
Cov(Y_1^2, Y_2) &= \cos^2\theta \sin\theta E(u_1^3) + \cos\theta \sin^2\theta E(u_2^3), \\
Cov(Y_1^3, Y_2) &= \cos^3\theta \sin\theta [E(u_1^4) - 3] - \cos\theta \sin^3\theta [E(u_2^4) - 3], \\
Cov(Y_1^2, Y_2^2) &= \cos^2\theta \sin^2\theta [E(u_1^4) + E(u_2^4) - 6], \\
Cov(Y_1^2, Y_2^3) &= \cos^2\theta \sin^3\theta E(u_1^5) + \cos^3\theta \sin^2\theta E(u_2^5) \\
&\quad + E(u_1^3)[3\cos^4\theta \sin\theta + 6\cos^2\theta \sin^3\theta + \sin^5\theta - \sin^3\theta] \\
&\quad + E(u_2^3)[3\cos\theta \sin^4\theta - 6\cos^3\theta \sin^2\theta + \cos^5\theta - \cos^3\theta], \\
Cov(Y_1^3, Y_2^3) &= \cos^3\theta \sin^3\theta [E(u_1^6) - E(u_2^6)] \\
&\quad + [E(u_1^4) - E(u_2^4)][3\cos^5\theta \sin\theta - 9\cos^3\theta \sin^3\theta + 3\cos\theta \sin^5\theta] \\
&\quad + E(u_1^3)E(u_2^3)[9\cos^2\theta \sin^4\theta - 9\cos^4\theta \sin^2\theta] \\
&\quad - [E(u_1^3) - E(u_2^3)] \cos^3\theta \sin^3\theta.
\end{aligned}$$

The closed-form expressions can be used to characterize the distributions of sources u_1, u_2, for which a given power covariance is not-informative, i.e. equal to zero for any value of θ.

On-Line Appendix 2

Additional Assumptions

The additional assumptions given below complete Assumption A.1 and are sufficient to derive Propositions 1 and 2 by applying the results in Shi, Shum (2015).

Additional Assumptions AA:

i) The parameter space Θ is compact with a non-empty interior.

ii) The function $\xi(\theta)$ is twice continuously differentiable on the interior of the parameter space

iii) The closure of the interior of the implied identified set is such that:
$cl[int\Theta_0^*(f_0)] = \Theta_0^*(f_0)$.

iv) The rank of $\frac{\partial\gamma(\theta)}{\partial\theta'}$ is constant on the interior of the implied identified set.

Condition AA(ii) implies the first part of Assumption (4) in Theorem 2.1 of Shi, Shum (2015) and corresponds to the assumption in Andrews et al. (2004). Condition AA iv) is the second part of their Assumption (4).

It means that the dimension of the identified set considered as a manifold exists, i.e. all its tangent spaces have the same dimension. In the simple regression model $Y - X\theta = \epsilon$, θ is not identifiable if the explanatory variables are strictly collinear, the identifying set for θ is a vector space, in particular a manifold of a dimension equal to *dim null* $(X'X)$. Then, this assumption is satisfied.