Positional Momentum and Liquidity Management; A Bivariate Rank Approach

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Abstract

This paper introduces a new positional momentum management strategy based on the expected ranks of asset returns and trade volume changes predicted from a bivariate Vector Autoregressive (VAR) model. The new method is applied to panel data of 1330 stocks traded on the NASDAQ between 1999 and 2016. The new prediction-based positional momentum strategy with bivariate ranks is shown to outperform the standard momentum strategies and the equally weighted portfolio. Next, the method is extended to positional liquidity management of portfolios of stocks with the largest gains or declines in trading volumes. These positional liquid portfolios are shown to produce even higher average and cumulative returns over various holding times than the positional momentum portfolios based on bivariate ranks.

Keywords: Positional Momentum Strategy, Gaussian Ranks, Panel VAR Model, Positional Momentum Portfolio, Positional Liquidity Portfolio.

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1 Introduction

In the financial literature and practice, the mean-variance portfolio is usually considered as the benchmark for portfolio management. At each time t, the mean-variance allocation is derived by maximizing the expected future portfolio returns adjusted for conditional risk, which is measured by the volatility. The conditional expected return and volatility are usually computed from the lagged expected returns. It is known that this optimal strategy introduced by Markovitz (1952) can have poor performance in practice, especially when the number of assets in the portfolio is large. This is due to inaccurate estimation of the inverse of the volatility matrix that affects the estimated portfolio allocations and to highly erratic evolution of the mean-variance portfolio allocation, which requires high turnover and increases the associated trading costs [Fisher and Statman (1997), and Leland (1999)]. This explains why other portfolio managements can compete with the mean-variance management, some of which being naive, such as the equally weighted portfolio management, for example [1].

This paper is focused on the momentum (or contrarian momentum) strategies. The basic positional momentum strategy consists in ranking the asset returns at time t and then building an equally weighted portfolio from the top alpha-percentile of all assets. The value of alpha is fixed to get the desired top percentile, such as the fifth top percentile, for example. The contrarian positional momentum strategy builds an equally weighted portfolio from the lower alpha percentile. These strategies, unlike the mean-variance approach, provide stable portfolio allocations over a given investment horizon, due to the discretization that underlies the rankings[2]. They also lead to better ex-post realized Sharpe performance than the ex-post realized Sharpe performance of the mean-variance portfolio as documented in Jegadeesh and Titman (1993)[3].

The aim of this paper is to extend the class of positional momentum strategies in three

[1]Demiguel, Garlappi and Uppal (2009) showed that equally weighted portfolio beats the sample-based mean-variance model in terms of Sharpe ratio, certainty-equivalent return, or turnover.

[2] There is substantial evidence of profitability of the momentum portfolios (Rouwenhorst 1998; Grundy and Martin 2001; Chordia and Shivakumar 2002; Griffin, Ji and Martin 2003; Karolyi and Kho 2004 and Barroso and Santa-Clara 2015).

[3]There is a large body of research that documents momentum at the level of individual stocks (Jegadeesh 1990; Jegadeesh and Titman 1993; Moskowitz, Ooi, and Pedersen 2012), at the level of industries (Moskowitz and Grinblatt 1999), in size and book-to-market portfolios (Lewellen 2002), and at the level of currencies or corporate bonds (Burnside, Eichenbaum and Rebelo 2011; Menkhoff, Sarno, Schmeling and Schrimpf 2012 and Jostova, Nikolova, Philipov and Stahel 2013).

respects. First, the ranks of asset returns and the ranks of trade volume changes are used jointly. The motivation for this approach stems from the empirical evidence documented in financial literature, which suggests that the trade volumes provide additional information and help predict future returns [4]. Among others, Lee and Rui (2002) described the dynamic relations (causal relations, the sign and magnitude of the effects) between stocks' trading volume and their returns on three markets (New York, Tokyo, and London). I show that similar significant dynamic relations exist between the ranks of returns and volume changes.

Second, I replace the positional momentum portfolio based on the current observed ranks by the positional momentum portfolio based on the expected future ranks. The future ranks of return are predicted from both ranks of return and volume changes. This extends the work by Gagliardini. et al., (2016) who introduced the approach involving the predicted future return rank, called the expected positional momentum strategy.

The search for returns with high future ranks (or high Sharpe performance) does not protect the investor from future high liquidity risk. After building a future return-optimal portfolio, the investor may end up with an illiquid portfolio. The third contribution is a new positional liquid portfolio that contains assets that display the highest (resp. lowest) past or future expected changes in trade volumes[5].

My empirical results show that the expected positional portfolios based on the return ranks predicted from their own past values and the past ranks of trade volume changes outperform the expected positional momentum portfolios of Gagliardini, et al. (2016) as well as the standard momentum strategies [see. e.g. Jegadeesh and Titman (1993),(2001), Hellstrom (2000), Arena. et al., (2008))] and the equally weighted portfolio in terms of average monthly returns and cumulative returns over investment horizons less or equal to 3 months or larger or equal to one year. Moreover, the positional liquid portfolios of stocks introduced in this paper provide even better outcomes in terms of average and

[4]Karpoff in 1987 by using Bivariate Regression model, VECM Model, VAR, IRF and Johansen's Co-integration test, showed a bi-directional causality between trading volume and stock return volatility; Gallant, Rossi and Tauchen 1992 studied the semi nonparametric estimation of the joint density of current price change and volume conditional on past price changes and volume; Chordia and Swaminathan 2000 found that, the trade volume is a significant determinant of the lead-lag patterns observed in stock returns; Arena, Haggard and Yan 2008 showed the existence of a positive time-series relation between momentum returns and aggregate idiosyncratic volatility and Pathirawasam 2011 revealed that the stock returns are positively related to the contemporary change in trading volume

[5]A positive association between trading activity and volume is documented in Demsetz (1968). Also see Chordia, Roll and Subrahmanyam (2000) and Barclay and Hendershott (2004). Johnson (2008) shows that volume is positively related to the variance of liquidity or liquidity risk.

cumulative return at all considered horizons. The strategy that selects stocks with high past or expected volume changes leads to the highest average and cumulative portfolio gains at most holding times. I examine the look-back and holding times of one month, as well as the holding times of 3,6 and 9 months and 1 to 8 years.

The ranks of return and volume changes are predicted from a bivariate panel Vector Autoregressive (VAR(1)) model. Empirical literature provides evidence, the vector-autoregressive (VAR) model can capture stock return serial dependence in a statistically significant manner[see Demiguel, Nogales and Uppal (2014)] [6]. In this paper, a panel VAR model is used to represent the dynamics of return and volume changes ranks transformed to bivariate Gaussian ranks. The main advantage of the panel VAR model is that it has an ability to capture both the dynamic and cross-sectional inter-dependencies. The panel VAR model is estimated from monthly returns and trade volumes ranks of 1330 stocks traded on NASDAQ between 1999 and 2016.

The paper is organized as follows: Section 2 provides the description and summary statistics of data on returns and trade volumes. Section 3 introduces the ranks of securities according to their relative returns and trade volume changes cross-sectionally in each month. Their transformation to Gaussian ranks is also explained. The panel VAR model of the bivariate Gaussian ranks and its estimation are discussed in Section 4. Section 5 explains how a given portfolio can be positioned among other stocks with respect to either return or changes in trade volume. In Section 6, I define the new expected positional momentum and liquidity strategies based on the predictions of ranks of returns and volume changes. We compare these strategies among themselves and with the equally weighted portfolio and the standard positional momentum. Section 7 concludes the paper. Additional results and proofs are provided in Appendices A and B.

2 Data Description

The panel data contains monthly returns and trade volumes of 1330 stocks traded on the NASDAQ from October 1999 to October 2016. These stocks have been chosen with

[6]Demiguel, Nogales and Uppal (2014) used the vector autoregressive (VAR) model to capture serial dependence in stock returns. In financial literature VAR models have been used for strategic asset allocation. For instance, Campbell and Viceira 1999 and 2002, Campbell, Chan, and Viceira 2003, Balduzzi and Lynch 1999, Barberis 2000.

respect to the daily average of Turnover/Traded Value of all NASDAQ stocks in 2015. The Turnover/Traded Value is defined as the total amount traded in the security's currency, which is calculated as the sum of numbers of shares times their corresponding prices. Stocks from the highest and the lowest $25th$ percentiles of Turnover/Traded Value have been selected. After deleting stocks with missing values between October 1999 and October 2016, we end up with 1330 stocks.

The trade volume of a security is defined as the total quantity of shares traded multiplied by the closing price of that security. To get the return and the changes in trade volume, the log return and the log volume changes are calculated as follows:

$$\begin{aligned} r_{it} &= ln(\frac{P_{it}}{P_{it-1}}) \qquad t = 1, \cdots, T;\ i = 1, \cdots, n, \\ tv_{it} &= ln(\frac{TV_{it}}{TV_{it-1}}) \qquad t = 1, \cdots, T;\ i = 1, \cdots, n, \end{aligned} \tag{2.1}$$

where P_{it}, P_{it-1} are the prices at time t and $t-1$, TV_{it}, TV_{it-1} are the trade volume at time t and $t-1$ and r_{it}, tv_{it} are the log return and log changes in trade volume of stock i at time t respectively[7]. The panel contains $n = 1330$ stocks observed over $T = 214$ periods of time (months)[8].

Figures 1 and 2 present the cross-sectional mean (Figure 1) and variance (Figure 2) of the return (r_t) and trade volume changes (tv_t) over time. In Figure 1 we see that the mean returns and mean volume changes do not show any seasonality or trend over time. According to these figures, the mean and variance of trade volume changes are more volatile than those of returns. From October 2000 to October 2004, the mean return varies a lot, and it takes a sharp downturn in July 2001 and May 2002. According to a report by the Cleveland Federal Reserve, this downturn can be viewed as part of a larger bear market or correction that began in 2000. The majority of specialists believe that this downturn could be a reversion to average stock market performance in a longer term context.

Indeed from 1998 to 2000, the NASDAQ rose almost 85% while before that time it had the annual growth of 10% to 15%. Afterward, the index dropped to the same level it would have achieved if the annual growth rate followed during 1987-1995 had continued

[7] The log changes in trade volume are also referred to as V-ROC (Value Rate-Of-Change). See Investopedia at www.Investipedia.com for definition and Podobink et.al (2009) for empirical study.

[8] At each date t, the information available is $I_t : \{r_{it}, tv_{it}, i = 1, \cdots, n\} \approx \{P_{it}, TV_{it}, i = 1, \cdots, n\}$.

up to 2002. On September 16, 2008, the mean of returns reached its lowest value. The reason was the massive failures of financial institutions in the United States, due primarily to exposure to packaged sub-prime loans and credit default swaps issued to insure these loans and their issuers, which rapidly devolved into a global crisis. These financial failures resulted in a number of bank failures in Europe and sharp reductions in the value of stocks and commodities worldwide.

Another major fall in stock market was the Black Monday of 2011, which refers to August 8, 2011, when the US and global stock markets crashed following the Friday night credit rating downgrade, by Standard and Poor's of the United States sovereign debt from AAA, or "risk free", to AA. After that in 2014 and 2016, the stock market had experienced the Bull Market. Retail investors, started to put money back in the market in 2013, allowing them to benefit from 2014 advance.

Figure 1: Cross-Sectional Mean of Return and Trade Volume Changes Over Time

In Figure 2 we observe that the variances of returns and of volume changes are above 0.01 over the period 1999-2016. After year 2012 both take values between 0.01 and 0.02. There have been periods when the variance was unusually high or unusually low. From the beginning of 2000 the return volatility was decreasing gradually until 2004, when it reached a more steady pattern. In 2008, the volatility of returns surged to more than 0.07, which is fairly high by historical standards, yet not without precedent. It remained high during the crisis of 2008-2010.

The cross-sectional variance of trade volume changes dropped from the average of 0.4 in 1999 to less than 0.2 in 2004 and fluctuated between 0.1 and 0.2 until 2016. In years 2008, 2009 and 2010 the crisis, it increased considerably in parallel to the variance of returns. After year 2014, both series of cross-sectional variances are less erratic and more smooth.

Figure 2: Cross-Sectional Variance of Return and Trade Volume Changes Over Time

2.1 Data Stationarity

To check the stationarity of returns and trade volume change series, the unit root test is applied separately to each panel [9]. In the panel unit root test literature, the null hypothesis is formally stated as H_0: "all of the series have one unit root". While the null hypothesis is common to all the panel unit root tests, the literature considers two different alternative hypotheses, H_1^a : "all of the series do not have unit root" and H_1^b : "at least one of the series has unit root". The alternative H_1^b has been criticized by some authors indicating that if H_0 is rejected we do not know which series have a unit root (Taylor and Sarno 1998). On the other hand, alternative H_1^a implicitly imposes a strong dynamic homogeneity restriction across the panel units (Levin et al. (2002), Im, Pesaran and Shin (2003)) while it may also has power in mixed situations where not all the series are stationary. In practice, those tests that consider the alternative H_1^a are less flexible and may be subject to the same criticism as those considering the alternative H_1^b. Given these two alternative hypotheses the panel unit root tests, can be obtained in two ways: Approach 1 is based on the t-ratio and approach 2 is based on the p-value. In the first case the alternative hypothesis is H_1^a and in the second one it is H_1^b (Maddala and Wu (1999) and Choi (2001)).

The tests based on the t-ratios are panel extensions of the standard Augmented Dickey-Fuller test (ADF) (Said and Dickey (1984)). There are two ways of applying these tests to panel data, either by pooling the units before computing a pooled test statistic (Levin et al.(2002)), or averaging the individual test statistics in order to obtain a group-mean test (Im et al.(2003)). On the other hand, the p-value combination tests are based on the idea that the p-values from N independent ADF tests can easily be combined to obtain a test of the joint hypothesis concerning all the N units. The advantages of the p-value combination approach are its simplicity and flexibility in specifying a different model for each panel unit and the ease in allowing the use of unbalanced panels. Table 1 provides the results of the stationarity tests for returns and trade volume changes.

[9] The assumption of cross-sectional independence is common in the literature. It allows for analytically derivation of the asymptotic distributions of the test statistics. In application to financial data, one may argue that some common systematic factors exist. In such a case, the asymptotic distributions change, but the test procedures remain consistent. The statistical adjustment of the tests for common factors is out of the scope of this paper.

Table 1: Stationarity Test for Return and Trade Volume Changes

In Table 1, Columns 1 and 2 show the outcomes of tests based on the t-ratio which were introduced by Levin and Lin (Lin and Chu (2002)) and Im et al.(2003). Columns 3 to 6 present the outcomes of tests based on the p-value by Maddala and Wu (1999), the modified p-test proposed by Choi (2001), the inverse normal test by Choi (2001) and the logit test by Choi (2001), respectively. All of these tests indicate that the data of monthly returns and trade volume changes are stationary.

3 Ranks of Returns and Trade Volumes Changes

3.1 Ranks

The updating of asset components and their quantities in a portfolio is a key problem of portfolio management. One of the most commonly used approaches is by ranking the stocks returns, and building the different momentum portfolios based on those ranks. The literature shows that a momentum strategy based on return ranks can outperform the mean-variance strategy based on returns [see Jegadeesh and Titman (1993), Moskowitz, Ooi and Pedersen (2012),Barroso and Santa-Clara (2015)]. This is usually explained by the fact that returns are more volatile than their ranks and the momentum strategy is less sensitive to extreme volatility and more robust. Many articles show that the rank of stock returns is more predictable than the individual returns. Indeed, Hellstrom (2000) found that the ranks can be predicted with a linear model and his empirical results show 63% hit rate for the sign of daily threshold-selected 1-day predictions.

In the literature, ex-post and ex-ante ranks have been introduced. The ex-post ranks are obtained by ranking all asset returns at time t from the smallest one to the largest one and then dividing their position by the total number of observations. Equivalently the ex-post return rank of asset i can be derived by inverting the empirical cross-sectional (CS) cumulative distribution function (cdf) of the returns at date t. In this case the observed ex-post ranks have the discrete empirical uniform distribution on $(1/n, 2/n, ..., 1)$. In the ex-ante ranks, the empirical cross-sectional cdf is replaced by its theoretical distribution function, so the ex-ante ranks have a cross-sectional uniform distribution on the interval $[0, 1]$.

The ex-ante ranks are predicted ranks. Since the ranks are defined up to an increasing transformation, I use the following transformation to build the Gaussian ranks [see Gagliardini et al.(2013)]. The Gaussian ranks are obtained from the corresponding uniform ranks by applying the quantile function of the standard Normal distribution. Standardizing the ranks ensures the cross-sectional Normal distribution of the rank variables. Let us consider two ex-post Gaussian ranks, one based on stock returns and the other one based on their trade volume changes. These rank series are related to the returns and trade volume changes by the following equations:

$$u_{i,t} = \Phi^{-1}(\hat{F}_t^r(r_{it})) \qquad t = 1, \cdots, T;\ i = 1, \cdots, n \tag{3.2}$$

$$v_{i,t} = \Phi^{-1}(\hat{F}_t^{tv}(tv_{it})) \qquad t = 1, \cdots, T;\ i = 1, \cdots, n \tag{3.3}$$

where $u_{i,t}$ is the Gaussian rank of return, $v_{i,t}$ is the Gaussian rank of trade volume changes, Φ is the cumulative distribution function (cdf) of the standard Normal , Φ^{-1} is its inverse that is the quantile function of the standard Normal and $\hat{F}_t^r, \hat{F}_t^{tv}$ are the cross-sectional empirical cumulative distribution functions of return and trade volume changes at date t , respectively .

To compute the Gaussian ranks, first I order all returns and trade volume changes from the highest to the lowest for each month and assign them absolute ranks from 1 to 1330 (since my sample includes 1330 stocks which are trading in NASDAQ). Next, I divide these ranks by the total number of stocks, which gives me the position of each stock in comparison to all observations in each month. That procedure provides the empirical cross-sectional cumulative distribution functions $\hat{F}_t^r, \hat{F}_t^{tv}$. Finally to transform these ranks to the Gaussian ranks, I find the equivalent quantile of the standard Normal distribution function for each position. For instance, if asset i has return probability equal to $\hat{F}_t^r = 0.90$, it means that, there are 90% of assets in the sample, which have smaller or equal returns at time t, and other 10% of assets have larger returns. Equivalently, if asset i has rank 0.90, there is a probability equal to 90% that the return at time t of any other asset is smaller or equal to the return of asset i. For this particular stock, the corresponding Gaussian rank is $u_{it} = 1.28$, that is the 90% quantile of the standard normal distribution function.

Information available at each time t is $J_t : \{u_{it}, v_{it}, i = 1, \cdots, n\}$ and $J_t \subset I_t$.

Figures 3 and 4 display the Q-Q plots for the two transformed observed ranks vectors u and v in October 2016. The figures confirm the cross-sectional Gaussian distribution of return ranks and trade volume change ranks. We see that, as expected, both ranks are cross-sectionally Normally distributed. In addition, the Shapiro normality tests applied to ranks u and v in each month, indicate that both ranks are Normally distributed cross-sectionally at each period of time.

Figure 3: QQ Plot and Histogram of u_i in October 2016

Figure 4: QQ Plot and Histogram of v_i in October 2016

3.2 Relation Between Ranks of Return and Trade Volume Change

3.2.1 Cross-Sectional Correlation

Let us study the relation between return and volume changes ranks to see if these series are cross-sectionally correlated. Figure 5 shows the time series of cross-sectional correlations between the two ranks [10], which is computed from the sample of 1330 stocks at each time t.

The cross-sectional correlation is fluctuating between -0.2 and 0.2. It reaches its highest value in 2001 (0.31). It turns and stays negative for one year between 2008 and 2009 (the crisis), until it falls to its lowest value in 2014 (-0.19). On average, the cross-sectional correlation is 0.02. It means that on average, the return rank has a positive correlation with the rank of trade volume changes and it increases (resp. decreases) when the rank of trade volume changes increases (resp. decrease).

[10] The sample cross-sectional correlation is $\frac{1}{n}\sum_i (v_{it} - v_{.t})(u_{it} - u_{.t}) / \sqrt{\frac{1}{n}\sum_i (u_{it} - u_{.t})^2 \frac{1}{n}\sum_i (v_{it} - v_{.t})^2}$, where $u_{.t} = \frac{1}{n}\sum_i u_{it}, t = 1, \cdots, T$ and $v_{.t}$ is defined accordingly.

Figure 5: Cross-Sectional Correlation Between u_{it} and v_{it}

3.2.2 Serial Correlation

For further insights, let us illustrate the serial correlation of the two series of ranks at various lags. As it is impossible to show the outcomes for all stocks in the sample, let us consider as an example the $S\&P500$ market index. Figure 6 shows the time series of ranks for monthly returns and trade volume changes of the $S\&P500$ during the post-crisis period of 2011-2016.

Figure 6: S&P.500 Return and Trade Volume Changes Ranks Over Time

We observe that, these two rank series of $S\&P500$ are fluctuating over time. In some periods of time they are moving in parallel (ex. May 2014-May2015) while in other periods they are moving in opposite directions (ex. December 2011-November 2012).

To illustrate the auto-correlation and cross-correlation functions between the two ranks series for the $S\&P500$, we plot the auto-correlation (ACF) and dynamic cross-correlation function (CCF) in Figures 7,8 and 9. Figure 7 shows significant auto-correlation at the first lag in u_t. This finding is consistent with the negative sign of return correlation reported in Jagadeesh (1990). It means that we can use the last month's return rank to predict the current rank of return. We also see same negative auto-correlation in ranks of trade volume changes at the first lag. Hence, we can use last month's rank of trade volume changes to predict the current one.

Figure 7: Auto-Correlation Function for u_t of S&P500

Figure 8: Auto-Correlation Function for v_t of S&P500

Figure 9: Cross-Correlation Function Between u_t, v_t of S&P500

Figure 9 shows the CCF of the ranks of returns and trade volume changes of $S\&P500$ as a proxy of the market. It shows a significant negative contemporaneous correlation between the two series of ranks of return and trade volume for $S\&P500$. In addition, it shows significant cross-correlations at the first and fourth lags, which suggests that past information, may help us predict the future ranks.

To illustrate the cross-correlations at lag one in all stocks, Figures 10 and 11 show the histograms of the cross-correlations between u_t, v_{t-1} and v_t, u_{t-1} computed from all stocks in the sample.

Figure 10: Cross-Correlation Between u(t) and v(t-1)

Figure 11: Cross-Correlation Between v(t) and u(t-1)

In Figure 10, the cross-correlation coefficients takes values between -0.13 to 0.18. The mode of the cross-correlation distribution is about 0.04 which means that, on average, u_t are positively correlated with v_{t-1}. The mode of the distribution in Figure 11 is -0.15 which shows that on average v_t and u_{t-1} are negatively correlated. The cross-correlation coefficients range between -0.29 and 0.16.

4 The Cross-Sectional Gaussian Ranks Model

4.1 The Model

The positional portfolio strategy is about finding the optimal allocation based on the future position of all equities in the portfolio. To predict the future positions, we need to define a joint dynamic model of ranks of return and trade volume changes. According to the results presented in the previous section, the joint dynamics of the two rank series can be represented by a Vector Autoregressive model of order one (VAR(1)).

$$\begin{pmatrix} u_{it} \\ v_{it} \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \begin{pmatrix} u_{i,t-1} \\ v_{i,t-1} \end{pmatrix} + \Sigma^{1/2} \begin{pmatrix} e_{1,it} \\ e_{2,it} \end{pmatrix} \qquad t = 2, \cdots, T;\ i = 1, \cdots, n, \tag{4.4}$$

where $R = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$ is the matrix of autoregressive coefficients , Σ represents the error variance matrix, and the idiosyncratic disturbance terms $(e_{1it}, e_{2,it})$ are serially independent and identically (i.i.d) standard Normal distributed. The autoregressive matrix R is assumed to have eigenvalues with modulus less than one to ensure the stationarity of the process. Since the ranks are marginally standard Normally distributed, the joint variance of the ranks has the form $\begin{pmatrix} 1 & \eta \\ \eta & 1 \end{pmatrix}$. Thus, we need to impose a constraint on the error variance matrix Σ as follows:

$$\begin{pmatrix} 1 & \eta \\ \eta & 1 \end{pmatrix} = R \begin{pmatrix} 1 & \eta \\ \eta & 1 \end{pmatrix} R' + \Sigma \qquad where\ \ |\eta| < 1 \tag{4.5}$$

The diagonal terms are the variances of u_{it} and v_{it} and η represents the contemporaneous correlation between u_{it} and v_{it}. To estimate the VAR(1) model, let us rewrite it as follows:

$$\begin{pmatrix} u_{it} \\ v_{it} \end{pmatrix} = R \begin{pmatrix} u_{i,t-1} \\ v_{i,t-1} \end{pmatrix} + \Sigma^{1/2} \begin{pmatrix} e_{1,it} \\ e_{2,it} \end{pmatrix} \qquad t = 2, \cdots, T; \quad i = 1, \cdots, n. \tag{4.6}$$

where $\Sigma = \begin{pmatrix} 1-A & \eta - C \\ \eta - C & 1-B \end{pmatrix}$, where $A = \rho_{11}^2 + \rho_{12}^2 + 2\eta\rho_{11}\rho_{12}$, $B = \rho_{21}^2 + \rho_{22}^2 + 2\eta\rho_{21}\rho_{22}$ and $C = \rho_{11}\rho_{21} + \rho_{12}\rho_{22} + \eta\rho_{11}\rho_{21} + \eta\rho_{11}\rho_{22}$.

The parameters of model (4.6) are estimated by the maximum log likelihood with the following objective function that is maximized with respect to the autoregressive parameters and η as follows:

$$logL(R, \eta) = \sum_{i=1}^{N} \sum_{t=2}^{T} \Big\{ -log(2\pi) - \frac{1}{2} log(|\Sigma|) - \frac{1}{2} \Big(\begin{pmatrix} e_{1,it} \\ e_{2,it} \end{pmatrix}' \Sigma^{-1} \begin{pmatrix} e_{1,it} \\ e_{2,it} \end{pmatrix} \Big) \Big\} \tag{4.7}$$

after replacing in the objective function Σ by its expression given above. The VAR model parameters are estimated from the entire sample of 1330 stocks over the period of 1999 to 2016. Table 2 shows the results of the maximum likelihood estimation.

Table 2: Estimated VAR(1) Model

According to the empirical results, all coefficients of the model are strongly significant. The estimated signs of the autoregressive coefficients suggest that:

1) low ranks of past returns and high ranks of past volume changes tend to increase the current ranks of returns,

2) low ranks of past returns and low ranks of past volume changes tend to increase the current ranks of volume changes.

The contemporaneous correlation $\hat{\eta}$ between the error terms is positive. It means, if the rank of trade volume change is high, it instantaneous positively affects the rank of return and makes it higher in the current period[11].

An important characteristic of a VAR process is its stationarity. As stationary VAR model components time series with time-invariant means, variances, and covariance structure. In practice, the stationarity of an empirical VAR process can be analyzed by considering the companion form and calculating the eigenvalues of the coefficient matrix. The obtained eigenvalues for the VAR(1) model in equation (4.4) are -0.348 and -0.025. Since both eigenvalues are of modulus less than one, we can conclude that the system is stationary.

After estimating the VAR-model, we test whether the residuals obey the model's assumption. First, we check for the absence of serial correlation and next, we verify if the error process is normally distributed. The Durbin-Watson (DW) statistic is used to detect the presence of autocorrelation at lag 1 in the residuals (prediction errors) from a regression analysis. The null hypothesis H_0 in the DW test is that the errors are serially uncorrelated and the alternative hypothesis H_1 is the existence of a first order autoregressive process in error terms. The DW test applied to the residuals of model (4.6) does not reject the null hypothesis against the alternative, which indicates that there is no autocorrelation

[11]This finding acknowledges the fact that we observed in Section 3.2.1 since the average cross -correlation between ranks of return and trade volume is 0.02.

in the residual terms [see Appendix A.1]. The cross-sectional and serial Normality of the residuals are illustrated in Appendix A.2.

Given that the sampling period is long, one can be concerned about the stability of the estimated parameters. To examine the fit of the model over the long sampling period, I computed the fitted values of the observed ranks. I also computed the time series of the coefficients, which are obtained by re-estimating the model (equation (4.6)) by rolling with the window of 109 months ($\simeq$9 years).

Figures 12 and 13 show the time series of the coefficients estimated by rolling over the period: March 2008-March 2016. We can see that there is some variation in $\hat{\rho}_{11}$, which is more pronounced than that in $\hat{\rho}_{12}$. Coefficients $\hat{\rho}_{11}$ varies between -0.015 and -0.005, $\hat{\rho}_{12}$ varies between 0 and -0.01, $\hat{\rho}_{21}$ fluctuates between -0.015 and -0.010 and $\hat{\rho}_{22}$ varies around -0.18.

Figure 14 shows the time series of $\hat{\eta}$ (contemporaneous correlation between u and v) from the rolling estimation over 109 months. As we can see, $\hat{\eta}_t$ shows a downward trend. From 2008 to 2009 it decreases from above 0.2 to slightly less than 0.2 until the end of 2010. Later on, $\hat{\eta}$ shows no changes until the beginning of 2012. After 2012, it decreases gradually to zero.

Figure 12: Time Series of $\hat{\rho}_{11}, \hat{\rho}_{12}$

Figure 13: Time Series of $\hat{\rho}_{21}, \hat{\rho}_{22}$

Figure 14: Time Series of $\hat{\eta}$

Given the slight variation of the parameters and to accommodate the period of crisis 2008-2010, I use henceforth the rolling estimation of VAR(1) model over a window of 109 months between October 2008 and October 2016. Over that period I update the parameters estimates every month.

5 Positional Portfolio

This Section explains how a given portfolio can be positioned among other stocks on the market with respect to either returns or trade volume changes. Later on, it will be assumed that there is no short sell, i.e. the numbers of shares of stocks included in the portfolio are non-negative.

5.1 Portfolio Return and Portfolio Activity

Let us first consider a portfolio of n stocks. The numbers of shares of these stocks are $\alpha_i,\ i = 1, \cdots, n$. These quantities are non-negative by the no short sell condition. At time t the prices are $P_{i,t},\ i = 1, \cdots, n$ and the dollar values of trade quantities are $TV_{i,t},\ i = 1, \cdots, n$. Then, the total value and total trade value for this portfolio are:

$$\bar{P}_t(\alpha) = \sum_{i=1}^{n} \alpha_i P_{i,t}, \qquad TV_t(\alpha) = \sum_{i=1}^{n} \alpha_i TV_{i,t}, \tag{5.8}$$

respectively. Thus, the changes between t and $t+1$ are:

$$\frac{\bar{P}_{t+1}(\alpha)}{\bar{P}_t(\alpha)} = \frac{\sum_{i=1}^{n} \alpha_i P_{i,t+1}}{\sum_{i=1}^{n} \alpha_i P_{i,t}} \tag{5.9}$$

$$\frac{TV_{t+1}(\alpha)}{TV_t(\alpha)} = \frac{\sum_{i=1}^{n} \alpha_i TV_{i,t+1}}{\sum_{i=1}^{n} \alpha_i TV_{i,t},}, \tag{5.10}$$

These changes can be rewritten as:

$$\frac{\bar{P}_{t+1}(\alpha)}{\bar{P}_t(\alpha)} = \sum_{i=1}^{n} \left(\frac{\alpha_i P_{i,t}}{\sum_{i=1}^{n} \alpha_i P_{i,t}} \frac{P_{i,t+1}}{P_{i,t}} \right) \equiv \sum_{i=1}^{n} \beta_{i,t}^{r} \frac{P_{i,t+1}}{P_{i,t}} \tag{5.11}$$

$$\frac{TV_{t+1}(\alpha)}{TV_t(\alpha)} = \sum_{i=1}^{n} \left(\frac{\alpha_i TV_{I,t+1}}{\sum_{i=1}^{n} \alpha_i TV_{i,t}} \frac{TV_{i,t+1}}{i,t} \right) \equiv \sum_{i=1}^{n} \beta_{i,t}^{tv} \frac{TV_{i,t+1}}{TV_{i,t}} \tag{5.12}$$

Equations (5.11)-(5.12) provide the aggregate formulas of changes in prices and trade volumes, respectively. These allocations are:

a) the allocation in number of shares: $\alpha_i,\ i = 1, \cdots, n$, or

b) the allocation in capitalization: $\beta^r_{i,t},\ i = 1, \cdots, n$, or

c) the allocation in dollar weighted by the activity: $\beta^{tv}_{i,t},\ i = 1, \cdots, n$.

All the above allocations are non-negative, by the no short sell assumption.

5.2 How to Position a Portfolio

In order to position a portfolio with no short sell, we use the aggregation formulas of Section 5.1. To make the link with the definitions of returns and changes in trade volume defined in the previous section, we also apply the aggregation formulas by substituting the geometric return (change) by their arithmetic counterpart.

This approximation is valid at the first order, as long as the changes are not too large. Thus, we can position a given portfolio among other assets by first defining the return (resp. activity) of that portfolio as:

$$r_t(\beta^r_t) = \beta^r_t r_t = \sum_{i=1}^{n} \beta^r_{i,t} r_{i,t} \tag{5.13}$$

$$tv_t(\beta^{tv}_t) = \beta^{tv}_t tv_t = \sum_{i=1}^{n} \beta^{tv}_{i,t} tv_{i,t} \tag{5.14}$$

Next, we deduce its position with respect to returns and changes in trade volumes:

$$u_t(\beta^r_t) = \Phi^{-1} F^r_t \Big[\sum_{i=1}^{n} \beta^r_{i,t} (F^r_t)^{-1} \Phi(u_{i,t}) \Big], \tag{5.15}$$

$$v_t(\beta^{tv}_t) = \Phi^{-1} F^{tv}_t \Big[\sum_{i=1}^{n} \beta^{tv}_{i,t} (F^{tv}_t)^{-1} \Phi(v_{i,t}) \Big], \tag{5.16}$$

where the c.d.f. F^r_t and F^{tv}_t are derived with respect to the universe of the n stocks considered.

6 Expected Positional Momentum Strategies

This section introduces the positional momentum portfolio management based on either the expected ranks of return or expected ranks of volume changes conditional on their past, and examines the comparative performance of the proposed portfolios. The standard positional momentum strategy consists in adjusting the portfolio by buying stocks or other securities with high observed past returns and selling stocks with poor observed past returns. The expected positional momentum strategy, introduced by Gagliardini et. al. (2016) extends that methodology by adjusting the portfolio so that at each time t it contains assets with high expected return ranks, which are forecasted out-of-sample from a univariate autoregressive model of return ranks [Appendix B].

In this paper, the momentum positional portfolio based on the expected return ranks is adjusted at each time t, conditional on the past return and volume change ranks. In our study, the expected ranks are forecasted out-of-sample from the bivariate VAR model (equation 4.6) of return and volume change ranks at each time t. Below, we present momentum portfolios based on the expected return ranks (Section 6.1) and the expected trade volume change ranks (Section 6.2).

In the literature, it has been observed that stocks reverse in returns at short monthly horizons (see e.g. Jegadesh (1990), Avramov et. al. (2006)) likely due to overreaction of some investors to news (De Bondt and Thaler (1985)). Therefore, we also consider the positional reverse momentum portfolios and positional reverse liquid portfolios which contain stocks with low expected return ranks and low expected volume change ranks, respectively.

In the positional strategy, it is important to define the investment universe which can be different than the positional universe. The investment universe is a set of assets potentially introduced in the portfolio, while the positional universe is a set of assets that has been used to define the ranks. For instance, for a fund manager, the investment universe may be a fraction of the stocks, whereas the positioning universe can be the set of all stocks which are trading in the stock market. This study, considers the investment universe equivalent to the positional universe (as in Section 5), which contains 1330 stocks returns and trade volumes in the balanced panel from NASDAQ market from 1992 to 2016.

6.1 Positional Momentum Strategies

This section examines the performance of the momentum strategy based on the expected return ranks. It also compares the proposed methodology based on the $VAR(1)$ model

(4.6) with the positional momentum strategy introduced by Gagliardini et. al. (2016) which is based on predicted return ranks from the AR(1) model (see Appendix B). All coefficients in both the VAR(1) and AR(1) models are estimated monthly and updated monthly by using a rolling window of 9 years of data.

6.1.1 Definition of The Strategies

The following strategies are considered to compute portfolios with monthly adjustments of asset allocations and equal look-back periods of one month over the period 2008 to 2016:

1) **The Expected Positional Momentum Strategy (EPMS)**

This strategy selects an equally weighted portfolio of stocks with the 5% highest expected return ranks in each month. The expected ranks at time t are estimated from one-step ahead forecasts for return ranks at time $(t-1)$ from, a) the bivariate VAR(1) model (equation (4.6)), b) the univariate AR(1) model (in Appendix B).

2) **The Expected Positional Reverse Strategy (EPRS)**

This strategy is similar to the EPMS except for including stocks with the 5% lowest expected return ranks at time t, which are estimated as one-step ahead forecasts from return ranks at time $t-1$ from a) the bivariate VAR(1) model (equation (4.6)), b) the univariate AR(1) model (in Appendix B).

3) **Equally Weighted Portfolio (EW)**

This portfolio is an Equally Weighted (EW) portfolio of all 1330 stocks at $(t-1)$. It is used in the performance study as the benchmark and a market portfolio proxy.

4) **The Positional Momentum Strategy (PMS)**

This is the standard strategy that selects at time $t-1$ an equally weighted portfolio including all stocks whose observed returns at time $(t-1)$ are in the upper 5% quantile of the CS (cross-sectional) distribution respectively. The Gaussian ranks of return of these stocks are such that $u_{i,t} \geq 1.64$.

5) **The Positional Reversal Strategies (PRS)**

This standard strategy selects an equally weighted portfolio including all stocks with the observed return ranks at time $(t-1)$ in the lowest 5% quantile of the CS distribution.

6.1.2 Return Performance of The Strategies

Figure 15 shows the time series of monthly portfolio returns generated by the above five strategies based on the $VAR(1)$ model. We observe a period of high volatility at the beginning of the sample due to the crisis. After 2008, the volatility decreases and the monthly returns on the EPMS portfolios are sometimes slightly below those on the EW and sometimes they are above. However as illustrated by the monthly returns in Table 3, the VAR based $EPMS$ strategy performs the best in the long term.

Table 3 presents statistics summarizing the monthly returns on the five positional portfolios over the sampling period $(2008-2016)$ including the expected positional strategies based on the $VAR(1)$ and $AR(1)$ models. On average, the VAR-based EPMS strategy provides the highest return and outperforms all other strategies.

Figure 15: Monthly Returns on Positional Momentum Portfolios

The average return on the equally weighted portfolio (EW) is slightly lower than on the PRS portfolio while being higher than on the PMS portfolio. This implies that the reversal portfolios based on the lower 5% past ranks of returns provide higher returns than the PMS which is based on upper 5% past ranks of returns. It means that the portfolio which is based on stocks that had lower ranks in the previous term, provides higher return than a portfolio of stocks with higher past ranks. Both the EPRS portfolios obtained from the AR and VAR models have lower average returns than the EW portfolio, while both provide positive average returns. However, the reversal portfolios based on the expected ranks of returns (EPRS) obtained from the bivariate (VAR) model provide higher average returns than the one, which is obtained, from the univariate (AR) model.

The last row of Table 3 provides the positional Sharpe ratio of these momentum portfolios. The Sharpe ratio obtained from the following formula:

$$SR = \frac{\bar{r}_p - r_f}{\sigma_p} \tag{6.17}$$

where $\bar{r}_p$ is the mean of the portfolio return, r_f is the risk free returns on the last date of holding the portfolios (October 2016) and σ_p is the standard deviation of the portfolio returns. In this paper, the time series of 10-year US Generic Government Treasury Bond is considered as a risk free-return. The last two lines in Table 3 show that, the bivariate VAR-based EPMS and EPRS portfolios have higher Sharpe ratios than the AR-based EPMS and EPRS. Both the highest average return and Sharpe ratio are on the EPMS obtained from the bivariate VAR model.

Table 3: Monthly Returns on Positional Momentum Portfolios

Figures 16 and 17 show the cumulative portfolio returns over time for all positional momentum portfolios with the inception date of January 2008, as computed from both the bivariate VAR model and the univariate AR model, respectively. The Figures correspond to self-financed portfolios, regularly adjusted according to the given momentum strategy up to 2016. In Figure 16, we observe that in early 2008, the cumulative returns on the PMS and EPRS are almost similar and higher than on other portfolios. The EW and the VAR-based EPMS portfolios are close and higher than the EPRS, while during the crisis period the EW outperforms both VAR-EPMS and VAR-EPRS portfolios. Between years 2009 and 2010, the cumulative return on the PMS is the highest. From 2010 to 2011 the EPMS, EPRS and PMS are almost equal. After year 2011, the VAR-based EPMS portfolio outperforms all other portfolios.

Figure 17 shows a similar pattern between 2008 to 2009. From 2009 to 2011, the AR-based EPRS and PMS portfolios provide the highest cumulative returns and AR-based EPMS and PRS portfolios provide the lowest cumulative returns. From 2012 until the end of 2013, the cumulative returns on all portfolios are very similar while for most of that time, the EPRS is better than other portfolios. Next, until September 2015, the cumulative return on the AR-based PRS is the highest and the return on the PMS is the lowest one.

Figure 16: Cumulative Returns on Positional Portfolios From 2008-VAR(1) Model

Figure 17: Cumulative Returns on Positional Portfolios From 2008-AR(1) Model

However, after mid-2015, the EW portfolio outperforms the AR-based EPMS and EPRS portfolios. By comparing these two graphs we see that after 2011, a positional momentum portfolio which is based on the VAR model predictions outperforms other positional momentum portfolios in terms of cumulative return.

Table 4 shows the cumulative return on October 2016 on the positional momentum portfolios with different inception dates. If one holds the positional momentum portfolios since January 1st, 2008, the VAR-based EPMS provides the highest cumulative return which is six times higher than EW cumulative return since 2014. Among the portfolios held from January 1st, 2010 the highest return is on the PRS which slightly exceeds the return on the EPMS.

Table 4: Cumulative Returns on Positional Momentum Portfolios

Among the positional portfolios held from January 1st, 2012, the VAR-based EPMS and the PRS are providing the highest cumulative returns. Among the portfolios held from January 1st 2014, the highest cumulative return is from the EW portfolio.

By comparing all the results in Table 4, we find that, the expected rank portfolios obtained from the bivariate VAR model provide significantly higher cumulative returns than those from the univariate AR model. Also, we find that the VAR-based EPMS provides the highest cumulative return on the positional momentum portfolio with a long holding period.

In Table 5, the first seven columns show the average cumulative returns on all five positional portfolios obtained by investing every month with one month look-back period. These are compared with the last two columns that show the positional momentum and reversal portfolios with the 3,6,9 and 12 months look-back periods. Four holding periods of 3,6,9 and 12 months respectively are considered. We observe that over the 3-month holding period, the VAR-based EPMS outperforms all other portfolios in terms of the Sharpe ratio and average cumulative return. Over the 6 and 9-month holding periods, the PRS based on 6 and 9 months look-back periods have the highest returns. Over the 12-month holding period, the VAR-based EPMS outperforms all other portfolios.

These results show that the VAR-based EPMS outperforms all other portfolios over short and long (3-and 12-month) holding times. We also see that without considering last two columns of different look-back periods, over 1-month look-back period at all different holding times, the VAR-based EPMS outperforms all other portfolios in terms of the Sharpe ratio and average cumulative return. It means that if every month an investor invest in a portfolio with a holding period of 3 or 6, 9 and 12 months[12], at the end of the sampling period (October 2016), the VAR-based EPMS will provide the highest average cumulative return. We also observe that the PRS outperforms the PMS at the holding periods of 3, 6, 9 and 12 months. This is consistent with the finding of Lehmann (1990) and Jegadeesh (1990) who find that stock returns exhibit strong reversals for short look-back periods (one to six months) [see also, Moskowitz, Ooi and Pedersen, 2012] and it is against the finding of Moskowitz and Grinblatt (1999), who believed that trading based on individual stock momentum appears to be highly profitable at intermediate horizons (the 6- to 12-month range).

Table 5: Rolling Cumulative Returns on Positional Momentum Portfolios

6.2 Expected Liquidity Positional Momentum Strategy

6.2.1 Definition of the Strategies

This section introduces the positional liquid portfolio management strategies that are defined below and named accordingly to the terminology introduced in the previous section, with the letter "L" for liquidity added to the acronyms [13]:

1) **The Liquid Expected Positional Momentum Strategy (LEPMS)**

This strategy selects at time t an equally weighted portfolio of stocks with the 5% highest expected ranks of trade volume changes in each month. The expected ranks at time

[12] We refer to this strategy as a rolling investment.

[13] Many micro-structure models suggest, it is easier to trade when the market is active, therefore linking the trade volume and liquidity.

t are estimated from one-step ahead forecasts from volume ranks at time $t-1$ obtained from a) the bivariate VAR(1) model (equation. (4.6)), b) the univariate AR(1) model for liquidity, as estimated by the ranks of volume changes.

2) **The Liquid Expected Positional Reverse Strategy (LEPRS)**

It selects at time t the 5% lowest expected ranks of trade volume changes of time t, which are estimated as one-step ahead forecasts from the ranks at time $t-1$ from a) the bivariate VAR(1) model (equation (4.6), b) the univariate AR(1) model for liquidity.

The LPMS and LPRS are the momentum portfolios directly defined from the past ranks of trade volume changes at time $t-1$. The EW portfolio remains an equally weighted portfolio of all stocks, as before.

6.2.2 Return Performance

Let us now compare the performance of the liquidity expected positional portfolios in terms of returns. Table 6, shows the summary statistics on the returns on the liquid expected positional portfolios over the sampling period. According to these results, all liquid expected positional portfolio strategies produce higher average return than the equally weighed portfolios (EW). The LPMS portfolio which is based on the highest trade volume ranks at time $(t-1)$ has the highest average return and the VAR-base LEPRS, which is based on the expected lowest volume ranks is close to the LPMS return.

Table 6: Returns on Positional Liquid Portfolios

By comparing the results in Table 6 to those in Table 3, we find that, the highest average returns are obtained from the positional liquid LPMS, the VAR-based LEPR strategy, followed by the VAR-based positional momentum EPMS strategy. Therefore, these two VAR-based portfolios outperform other positional strategy portfolios, in terms of average portfolio returns. This means that, on average the positional portfolios based on very liquid assets with either past strong increase in trading activity or a predicted strong decline in trading volume provide more return than those based on stocks with the highest returns. In terms of the Sharpe ratio, the LPMS and the VAR-based LEPRS have the highest Sharpe ratio respectively. According to these results, all liquid expected positional portfolio strategies produce higher average returns than the equally weighted portfolio.

As in the previous section, we can compare the positional liquid portfolios in terms of their cumulative returns. Figures 18 and 19 show the time series of cumulative return on the positional liquid portfolios held since 2008 and based on the VAR and AR models respectively.

Figure 18: Cumulative Returns on Positional Liquidity Portfolios From 2008 - VAR(1) Model

Figure 19: Cumulative Returns on Positional Liquidity Portfolios From 2008 - AR(1) Model

In both Figures we see parallel patterns for all portfolios. In 2008, all portfolios provide close cumulative return. Their cumulative returns dropped rapidly at the end of the year. From 2009 to the end of the sample period, the LPMS has the highest cumulative return while the LEPRS was slightly below. From 2009 to the beginning of 2014, the EW portfolio has higher cumulative return than the LPRS and the LEPMS. After 2014, it has lower cumulative returns than the LPRS.

Table 7 shows the cumulative returns on the positional liquid portfolios based on the VAR and AR models starting form different years. We see that, all positional liquid portfolios provide positive cumulative returns over the sampling period. Among the portfolios held from 2008, the highest cumulative return is on the LPMS. Over the holding period 2008-2016, the highest cumulative return is on portfolios with past strong increase in trade volume (LPMS) followed by the portfolio with predicted strong decline in trade volume (VAR-based) LEPRS. Over the holding period 2010-2016, the highest cumulative return is provided by a portfolio with a past strong decline in trade volume (LPRS). Over shorter perios of time (2012-2016 and 2014-2016), the highest cumualtive returns are provided by portfolios with the highest increase in trade volume predicted from the VAR model (LEPMS)

Table 7: Cumulative Returns on Positional Liquid Portfolios

By comparing the results from Table 7 with Table 4, we find that the positional liquid portfolios LPMS which is based on stocks with the past highest changes in trade volume outperforms other positional momentum portfolios in the long run (8 years). If an investor is planning to invest in long run he or she can benefit the most from holding the LPMS. In period of 6 years investment from 2010-2016 the LPRS followed by the VAR-based LEPMS outperform the PRS and the VAR-based EPMS respectively. Over the shorter period of investment for 4 and 2 years, the VAR-based LEPMS which is the portfolio of stocks with the highest increase in trade volume predicted from the VAR model outperforms all positional momentum portfolios in Table 4. These results show that if one is planing to invest over shorter time he or she can benefit the most by holding the VAR-based LEPMS.

In Table 8 the first seven columns display the average cumulative returns on all five positional liquid portfolios obtained by investing every month with one month look-back period. The last two columns show the cumulative returns for positional liquid portfolios based on the top and bottom 5% stocks, with 3,6,9 and 12 months look-back periods. These results are obtained from four holding periods of 3,6,9 and 12 months respectively. We see that over the 3-month holding period, LPMS outperforms all other portfolios in terms of the Sharpe ratio and average cumulative return. Over the 6-month holding period the LPMS has the highest return, followed by the VAR-based LEPRS and the LPMS with a 6-months look-back period. Over the 9-month holding period, the LPMS with a 9-month look-back period outperforms all other portfolios. Over the 12-month holding period, the LPMS and VAR-based LEPRS provide the highest returns respectively.

By comparing Table 8 with Table 5, we see that the LPMS with 1-month look-back period outperforms all positional momentum portfolios. Over the 6-month holding period, the LPMS outperform all positional momentum portfolios, while the PRS with 6-month look-back period provides the same cumulative returns as the VAR-based LEPRS and LPMS with 6-month look-back periods. Over the 9-month holding period the LPMS with the same look-back period outperforms all positional momentum portfolios. Over the 12-month holding period, the LPMS with 1-month look back period outperforms all other positional momentum portfolios.

Table 8: Rolling Cumulative Returns on Positional Liquid Portfolios

7 Conclusion

This paper introduced positional momentum and liquidity portfolio management strategies which are based on the expected bivariate ranks of returns and trade volume changes. The ranks of returns and of trade volume changes are transformed to Gaussianity by the quantile function, i.e. the inverse of the cumulative Normal distribution function. The expected ranks are predicted from a conditionally Gaussian vector autoregressive model of order one VAR(1), which represents their joint dynamics. The predicted ranks are used to build the Expected Positional Momentum/Reversal portfolios (EPMS and EPRS) of stocks with high/low ranked expected returns. For portfolio liquidity management, I introduce the Liquidity (Expected) Positional Momentum and Reversal portfolios (LEPMS, LPMS, LEPRS and LPRS) of stocks with high and low ranked (Expected) trade volume changes.

These allocation strategies were applied to an investment universe consisting of 1330 stocks which are traded on the NASDAQ market between 1999-2016. The empirical results show that the VAR-based positional momentum EPMS portfolios outperform the equally weighted portfolio, the traditional momentum portfolios and the EPMS portfolios with return ranks predicted from a univariate AR model. This finding suggests that the trade volume ranks helped to predict the return ranks and improved the positional portfolio performance.

The analysis of the positional liquid portfolios shows that, the portfolios of stocks with strong past or expected increase in trading volume provide return higher than any positional momentum portfolio (EPMS which is based on the highest expected returns) and the equally weighted portfolio. This interesting result shows that a positional portfolio of stocks with a past high increase in trading volumes outperforms a portfolio of stocks with high expected returns.

I also find that over the period $2008 - 2010$ (the crisis) the LEPRS portfolio obtained from the predictions of the VAR model provides higher average cumulative returns than the equally weighted and the EPMS portfolios. After the crisis, the LEPMS portfolio based on VAR model predictions provides higher cumulative return than other positional momentum portfolios.

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Table 1: Stationarity Tests for Return and Trade Volume Changes

Variables	*Levinlin*	*Ips*	*Madwu*	*Pm*	*Invnormal*	*Logit*
Returns	-565***	-539***	153322***	2065***	-377***	-1159***
TradeVolumeChanges	-473***	-561***	154848***	2086***	-380***	-1170***

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 1 provides the results of six stationarity tests for returns and trade volume changes. Columns 1 and 2 show the results of t-ratio-based stationarity tests of Levin and Lin (Lin and Chu (2002)) and Im et al.(2003), respectively. Columns 3 to 6 present the outcomes of p-value-based stationarity tests of Maddala and Wu (1999), the modified p-test of Choi (2001), the inverse normal test of Choi (2001) and the logit test of Choi (2001), respectively.

Table 2: Estimated VAR(1) Model

$Coefficients$	$Values$
ρ_{11}	-0.024***
ρ_{12}	0.010***
ρ_{21}	-0.024***
ρ_{22}	-0.354***
η	0.0390***

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 2 provides the parameters of the VAR model, estimated from the entire sample of 1330 stocks over the period 1999 to 2016, by the maximum likelihood (equation 4.7). All coefficients of the model are strongly significant. The estimates of autoregressive coefficients imply that returns' ranks are related negatively with their own past value, while they are related positively with the past value of trade volumes ranks. The ranks of trade volumes are related negatively with both past ranks of trade volumes and returns.

Table 3: Monthly Returns on Positional Momentum Portfolios

	EW	PMS	PRS	EPMS		EPRS	
				VAR	AR	VAR	AR
$Mean$	0.0042	0.0020	0.0043	**0.0064**	0.0037	0.0034	0.0022
$Standard\ Deviation$	0.0676	0.0798	0.1202	0.1054	0.1090	0.0770	0.0743
$Skew$	-2.1586	-1.0888	-1.654	-1.3467	-1.5703	-1.6173	-1.1998
$Kurt$	12.168	9.222	9.6130	8.444	9.6056	12.485	9.7066
$Sharpe\ Ratio$	0.0416	0.0067	0.0240	**0.0471**	0.0206	0.0258	0.0103

Table 3 presents the summary statistics of monthly returns on the positional portfolios over the sampling period September 2008 to October 2016 including the expected positional strategies based on the $VAR(1)$ and $AR(1)$ models. On average, the VAR-based EPMS strategy outperforms all other strategies in terms of the mean and Sharpe ratio. The average return of the VAR-based EPRS is higher than the AR-based EPRS but lower than the EW.

Table 4: Cumulative Returns on Positional Momentum Portfolios

	EW	*PMS*	*PRS*	EPMS		EPRS	
				VAR	*AR*	*VAR*	*AR*
From 2008	0.444	0.206	0.451	**0.667**	0.384	0.358	0.230
From 2010	0.646	0.136	**0.750**	**0.729**	0.716	0.412	0.151
From 2012	0.528	0.280	**0.630**	**0.630**	0.613	0.419	0.249
From 2014	**0.102**	-0.015	0.034	**0.047**	0.019	0.003	-0.073

Table 4 shows the cumulative returns on October 1st 2016 on holding the positional momentum portfolios from different inception dates. If one holds the positional momentum portfolios from January 1st, 2008, the VAR-based EPMS provides the highest cumulative return which is six times higher than EW cumulative return since 2014. Among the portfoios held from January 1st, 2010 the highest return is on the PRS which is slightly higher than on the EPMS.

Table 5: Rolling Cumulative Returns on Positional Momentum Portfolios

Holding Period	Look-Back=1 Month							Look-Back=Holding	
	EW	PMS	PRS	EPMS		EPRS		PMS	PRS
3 Months				VAR	AR	VAR	AR		
Mean	0.013	0.006	0.014	**0.021**	0.012	0.011	0.007	0.003	0.020
Standard Deviation	0.118	0.148	0.200	0.178	0.178	0.151	0.151	0.135	0.182
Sharpe Ratio	0.101	0.033	0.067	0.112	0.061	0.062	0.039	0.009	0.092
6 Months				VAR	AR	VAR	AR		
Mean	0.026	0.010	0.029	**0.042**	0.024	0.020	0.012	**0.044**	**0.056**
Standard Deviation	0.184	0.229	0.307	0.271	0.271	0.236	0.236	0.164	0.223
Sharpe Ratio	0.136	0.038	0.089	0.149	0.084	0.078	0.047	0.216	0.214
9 Months				VAR	AR	VAR	AR		
Mean	0.043	0.015	0.050	**0.068**	0.043	0.031	0.020	**0.077**	**0.125**
Standard Deviation	0.213	0.264	0.352	0.311	0.311	0.272	0.272	0.148	0.453
Sharpe Ratio	0.197	0.052	0.138	0.213	0.133	0.109	0.069	0.431	0.248
12 Months				VAR	AR	VAR	AR		
Mean	0.073	0.029	0.090	**0.110**	0.079	0.054	0.037	0.086	0.055
Standard Deviation	0.206	0.260	0.336	0.302	0.302	0.267	0.267	0.100	0.188
Sharpe Ratio	0.347	0.106	0.264	0.361	0.257	0.197	0.134	0.690	0.120

The first seven columns of Table 5 display the average cumulative returns on all five positional momentum portfolios obtained by investing every month with one month look-back period. The last two columns show the cumulative returns for positional momentum portfolios based on the top and bottom 5% stocks, with 3,6,9 and 12 months look-back periods. These results are obtained on four holding periods of 3,6,9 and 12 months respectively. We see that for the 3-month holding period, the VAR-based EPMS outperforms all other portfolios in terms of the Sharpe ratio and average cumulative return. For the 6 and 9 month holding periods, the PRS based on 6 and 9 months look-back periods have the highest return. For the 12 month holding period, the VAR-based EPMS beats all other portfolios.

Table 6: Returns on Positional Liquid Portfolios

	EW	*LPMS*	*LPRS*	LEPMS		LEPRS	
				VAR	*AR*	*VAR*	*AR*
Mean	0.0042	**0.0101**	0.0048	0.0045	0.0042	**0.0085**	0.0077
Standard Deviation	0.0676	0.0723	0.06862	0.0692	0.0698	0.0710	0.0717
Skew	-2.1586	-1.0298	-2.1103	-2.1448	-2.2811	-1.0657	-1.4560
Kurt	12.168	6.1592	12.742	12.940	13.539	6.2801	8.2578
Sharpe Ratio	0.0416	**0.1198**	0.0491	0.0442	0.0401	**0.1004**	0.0879

Table 6 shows the summary statistics for returns on the liquid positional portfolios over the sampling period (1999 to 2016). All liquid expected positional portfolio strategies produce higher average returns than the equally weighed portfolio (EW). The LPMS portfolio which is based on the highest trade volume ranks at time $(t-1)$ has the highest average return. The return on VAR-based LEPRS, which is based on the expected lowest volume changes ranks is very close to LPMS return.

Table 7: Cumulative Returns on Positional Liquid Portfolios

	EW	*LPMS*	*LPRS*	LEPMS		LEPRS	
				VAR	*AR*	*VAR*	*AR*
From 2008	0.444	**1.051**	0.501	0.469	0.442	**0.893**	0.807
From 2010	0.646	0.724	**0.819**	**0.809**	0.800	0.605	0.611
From 2012	0.528	0.617	0.707	**0.709**	0.658	0.553	0.545
From 2014	0.102	0.079	0.195	**0.204**	0.166	0.051	0.079

Table 7 presents shows the cumulative returns on October 19th, 2016 on holding the positional liquid portfolios from different inception dates. If one holds the positional momentum portfolios from January 1st, 2008, the LPMS provides the highest cumulative return. The VAR-based LEPRS has the next highest return. For the investment made in 2010, the highest cumulative return is on the LPRS. For the shorter holding times of 4 ans 2 years (from 2012 and 2014 respectively), the highest return are provided by the VAR-based LEPMS.

Table 8: Rolling Cumulative Returns on Positional Liquid Portfolios

Holding Period	Look-Back=1 Month							Look-Back=Holding	
	EW	*LPMS*	*LPRS*	LEPMS		LEPRS		*LPMS*	*LPRS*
3 Months				*VAR*	*AR*	*VAR*	*AR*		
Mean	0.013	**0.032**	0.016	0.015	0.014	**0.027**	0.024	0.019	0.017
Standard Deviation	0.118	0.131	0.127	0.128	0.129	0.129	0.127	0.117	0.125
Sharpe Ratio	0.101	0.235	0.114	0.106	0.098	0.202	0.183	0.129	0.107
6 Months				*VAR*	*AR*	*VAR*	*AR*		
Mean	0.026	**0.065**	0.032	0.030	0.028	**0.056**	0.049	**0.056**	0.053
Standard Deviation	0.184	0.209	0.199	0.199	0.201	0.205	0.199	0.169	0.133
Sharpe Ratio	0.136	0.304	0.155	0.143	0.133	0.266	0.241	0.283	0.335
9 Months				*VAR*	*AR*	*VAR*	*AR*		
Mean	0.043	**0.099**	0.052	0.049	0.047	0.086	0.076	**0.130**	**0.101**
Standard Deviation	0.213	0.246	0.231	0.231	0.235	0.241	0.231	0.276	0.157
Sharpe Ratio	0.197	0.399	0.222	0.207	0.193	0.350	0.325	0.423	0.563
12 Months				*VAR*	*AR*	*VAR*	*AR*		
Mean	0.0731	**0.146**	0.087	0.081	0.080	**0.126**	0.115	0.115	0.108
Standard Deviation	0.206	0.253	0.222	0.224	0.229	0.247	0.229	0.115	0.130
Sharpe Ratio	0.347	0.571	0.385	0.359	0.344	0.507	0.496	0.845	0.698

The first seven columns of Table 8 display the average cumulative returns on all five positional liquid portfolios obtained by investing every month with one month look-back period. The last two columns show the cumulative returns for positional liquid portfolios based on the top and bottom 5% stocks, for 3,6,9 and 12 months look-back periods. These results are obtained on four holding periods of 3,6,9 and 12 months respectively. We see that for the 3-month holding period, the LPMS outperforms all other portfolios in terms of the Sharpe ratio and average cumulative return. For the 6 month holding period the LPMS has the highest return, with 6-month look-back period. For the 9 month holding period the LPMS with the same holding period is the best portfolio. For the 12 month holding period, the LPMS and VAR-based LEPRS have the highest returns respectively.

Figure 1: Cross-Sectional Mean of Return and Trade Volume Changes Over Time

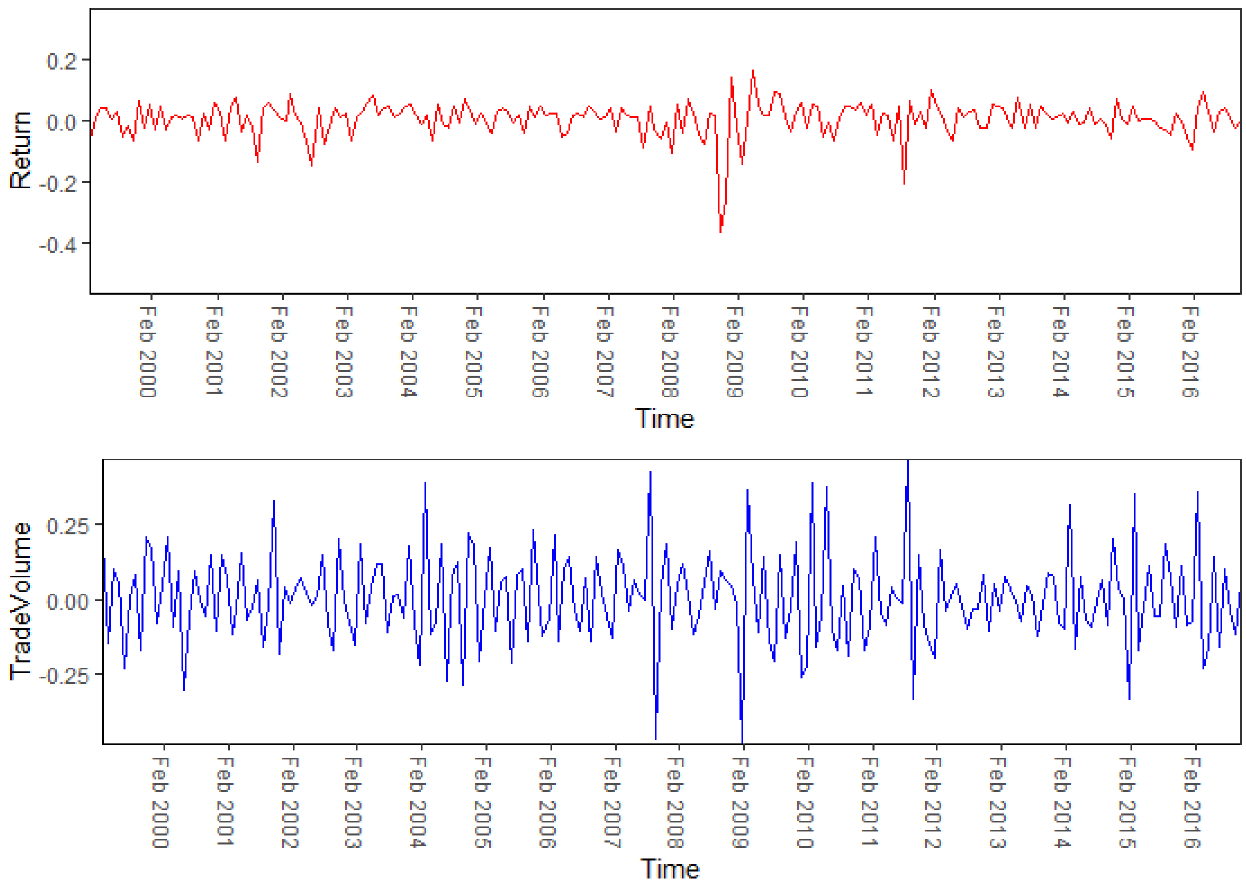


Figure 1 displays the cross-sectional mean of return (r_t) and trade volume changes (tv_t) from February 1999 to October 2016. The cross-sectional mean is computed monthly as the average of 1330 stocks' returns and trade volume changes traded on NASDAQ. The mean returns and mean volume changes do not show any seasonality or trend over time.

Figure 2: Cross-Sectional Variance of Return and Trade Volume Changes Over Time

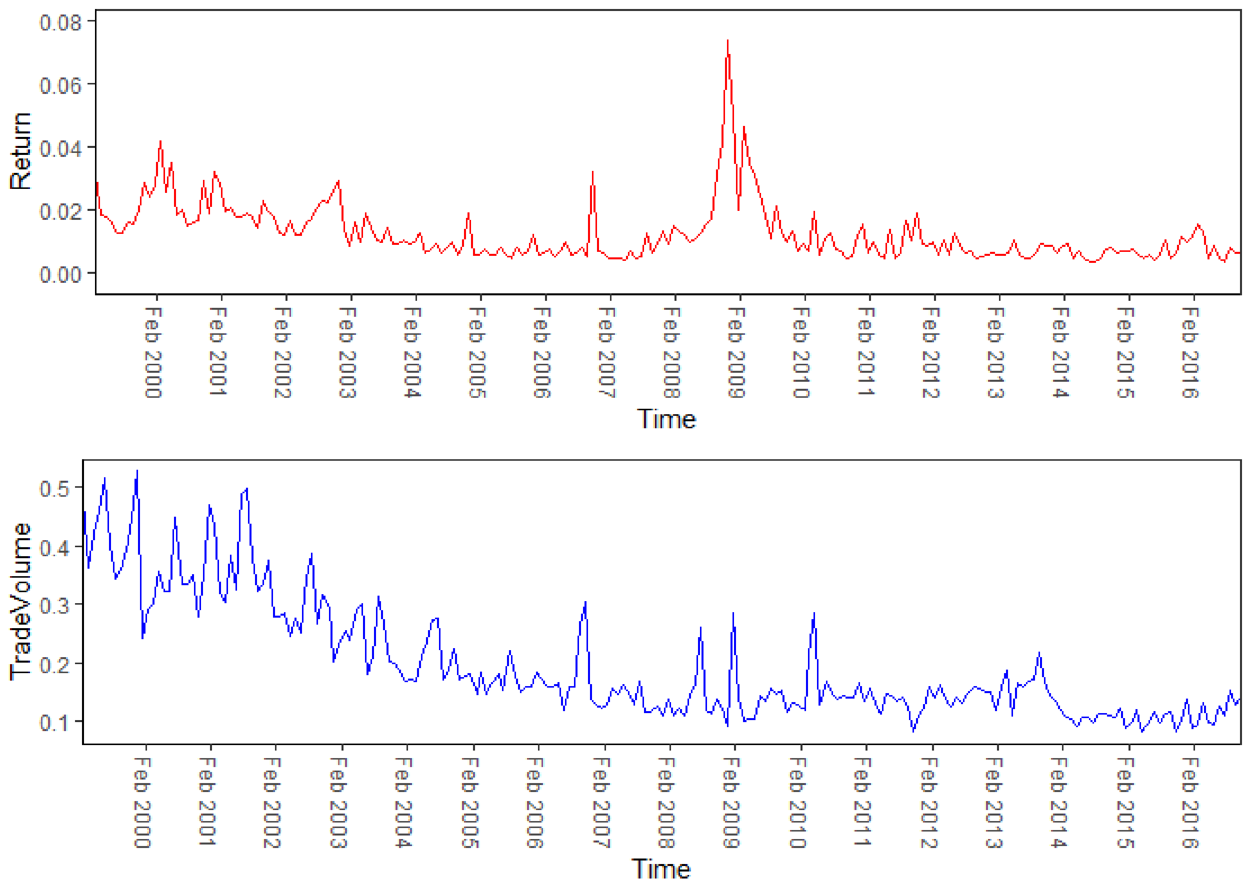


Figure 2 displays the cross-sectional variance of return (r_t) and trade volume changes (tv_t) from February 1999 to October 2016. The cross-sectional variance is computed monthly from 1330 stocks' returns and trade volume changes traded on NASDAQ. We see an episode of high return variance during the crisis 2008-2010 and high variance of volume changes prior to year 2005.

Figure 3: QQ Plot and Histogram of u_i in October 2016

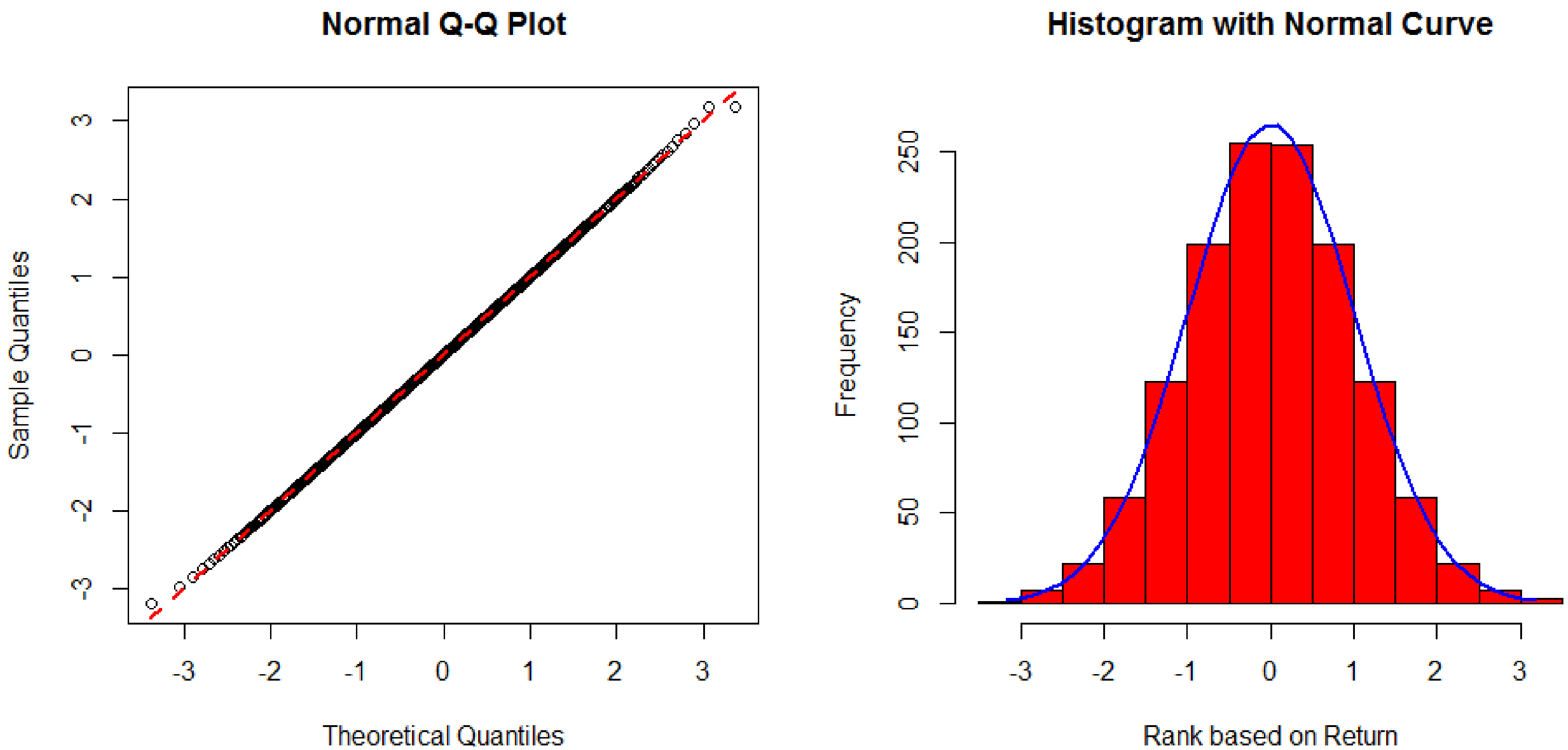


Figure 3 displays the Q-Q plot of the transformed return ranks u_i in October 2016. The figures confirm the cross-sectional Gaussian distribution of transformed return ranks.

Figure 4: QQ Plot and Histogram of v_i in October 2016

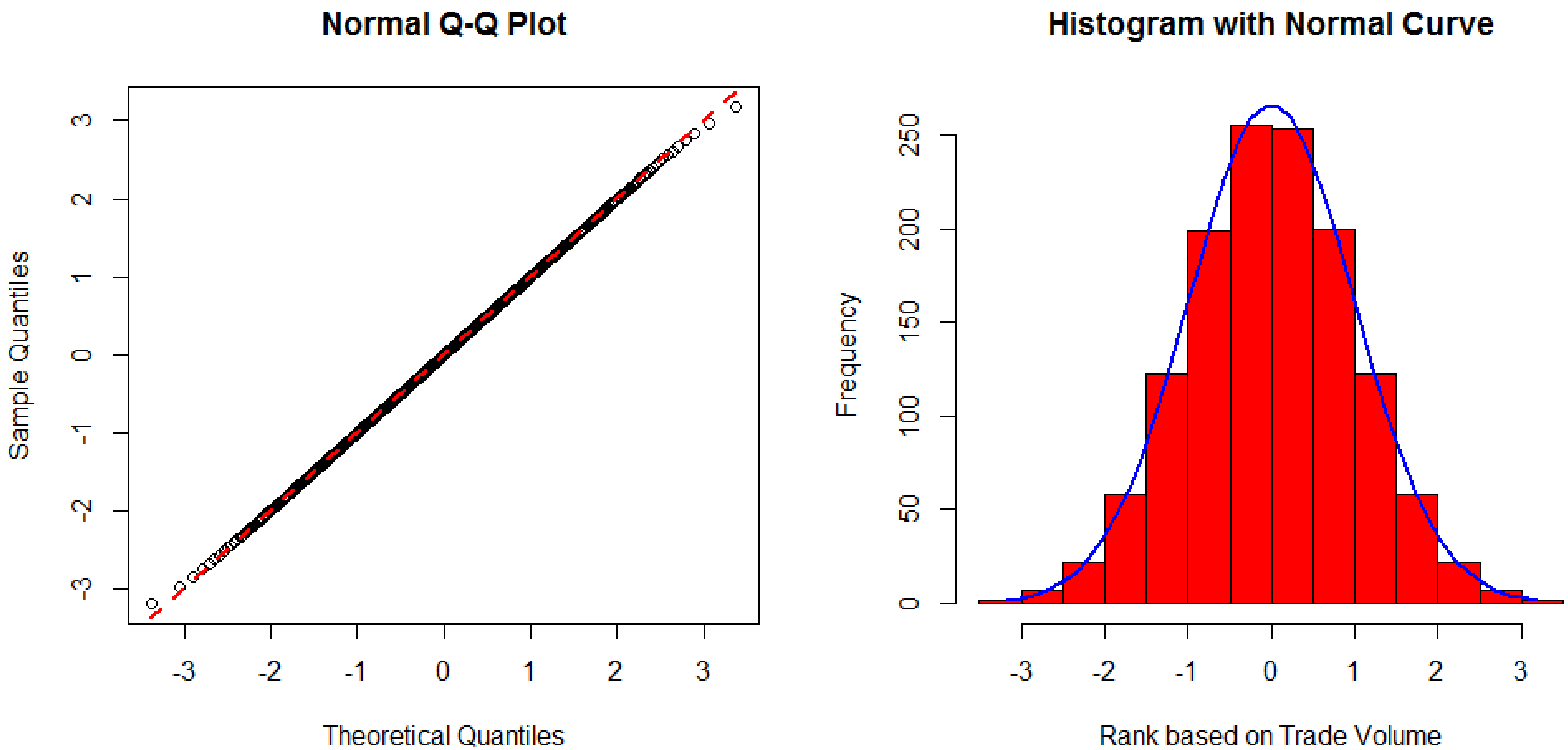


Figure 4 displays the Q-Q plot of the transformed trade volume changes ranks v_i in October 2016. The figures confirm the cross-sectional Gaussian distribution of transformed trade volume changes ranks.

Figure 5: Cross-Sectional Correlation Between u_{it} and v_{it}

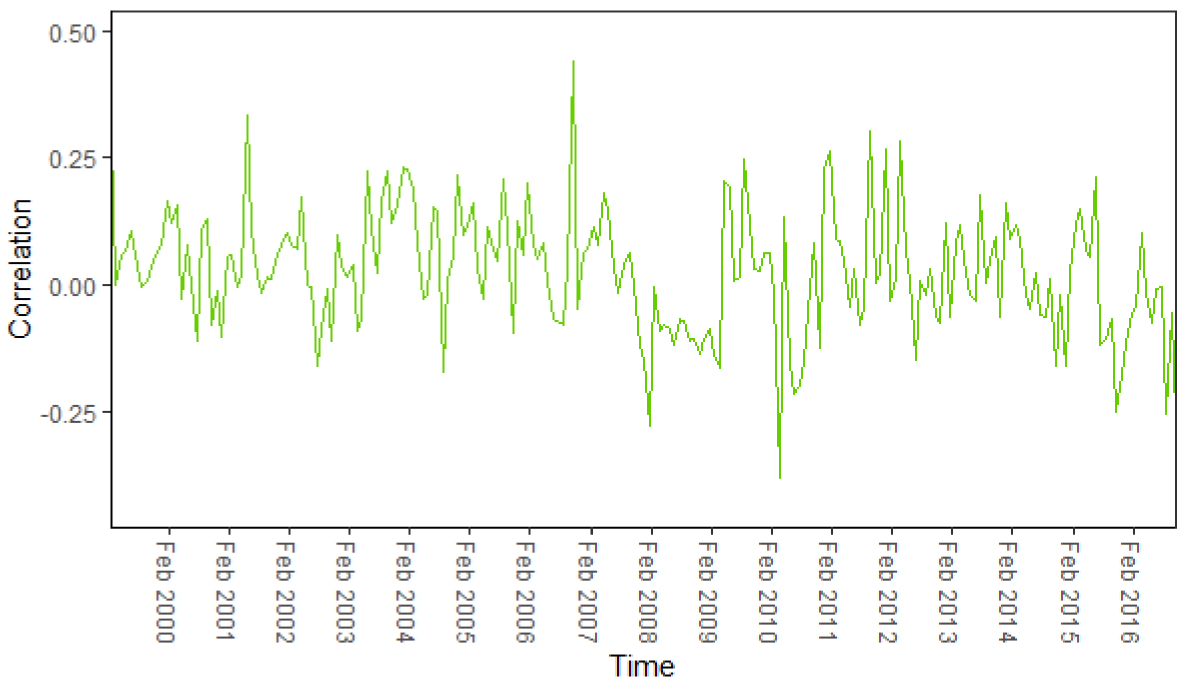


Figure 5 shows the time series of cross-sectional correlations between the return and trade volume changes ranks. The sample cross-sectional correlation is $\frac{1}{n}\sum_i(v_{it}-v_{.t})(u_{it}-u_{.t})/\sqrt{\frac{1}{n}\sum_i(u_{it}-u_{.t})^2\frac{1}{n}\sum_i(v_{it}-v_{.t})^2}$, where $u_{.t}=\frac{1}{n}\sum_i u_{it}, t=1,\cdots,T$ and $v_{.t}$ is defined accordingly and are computed from the sample of 1330 stocks at each time t.

Figure 6: S&P.500 Return and Trade Volume Changes Ranks Over Time

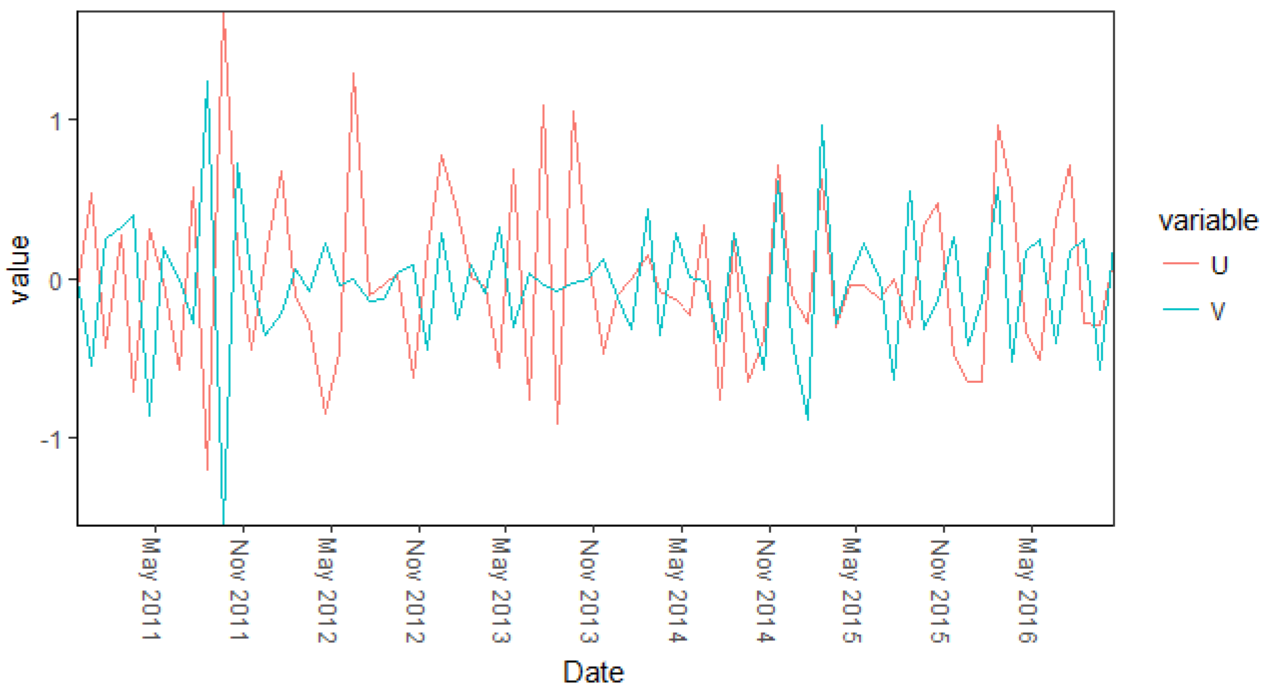


Figure 6 shows the time series of Gaussian ranks for monthly returns and trade volume changes of the $S\&P500$ during the post-crisis period of 2011-2016. These two rank series of $S\&P500$ are fluctuating over time. In some periods of time they are moving in parallel while in other periods they are moving in opposite directions (ex. December 2011-November 2012).

Figure 7: Auto-Correlation Function for u_t of S&P500

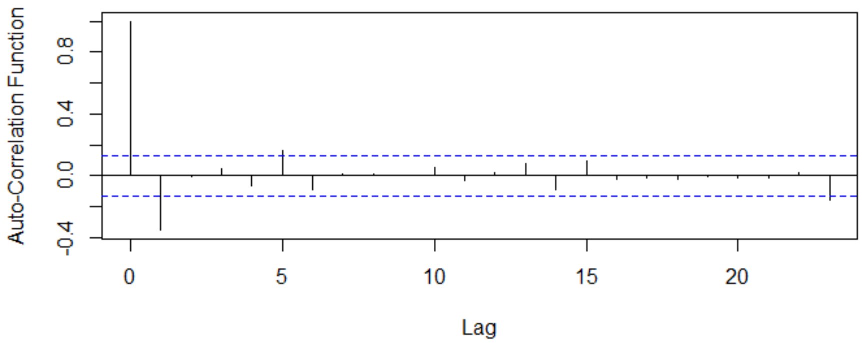
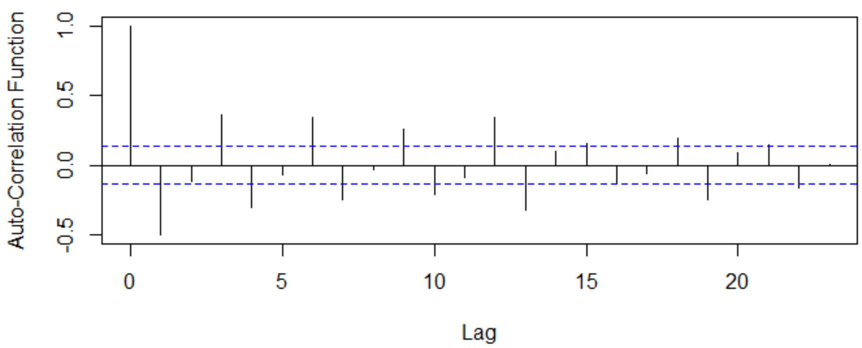


Figure 8: Auto-Correlation Function for v_t of S&P500



Figures 7 and 8 show the auto-correlation function (ACF) of u_t and v_t series for the $S\&P500$ as the index of the NASDAQ market. Figure 7 shows significant auto-correlation at the first lag in u_t. In Figure 8 we see negative auto-correlation at the first lag in ranks of trade volume changes.

Figure 9: Cross-Correlation Function Between u_t, v_t of S&P500

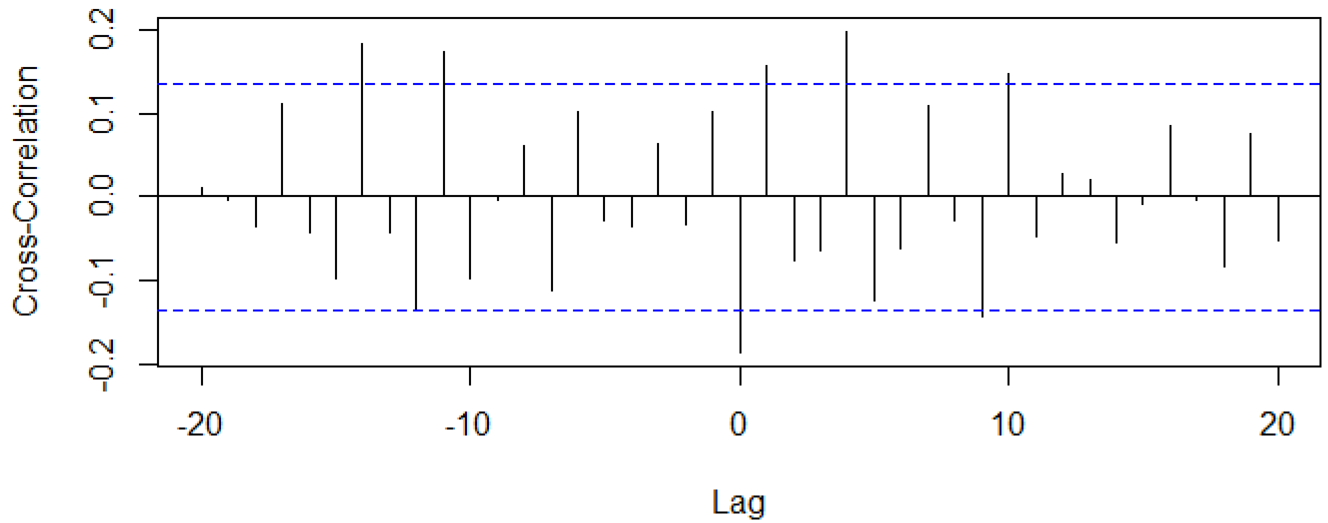


Figure 9 displays the cross-correlation functions (CCF) between the two ranks series (u_t, v_t) for the $S\&P500$ as a proxy of the NASDAQ market. The CCF shows a significant negative contemporaneous correlation between the two series of ranks of return and trade volume for $S\&P500$. In addition, it shows significant cross-correlations at the first and fourth lags.

Figure 10: Cross-Correlation Between u(t) and v(t-1)

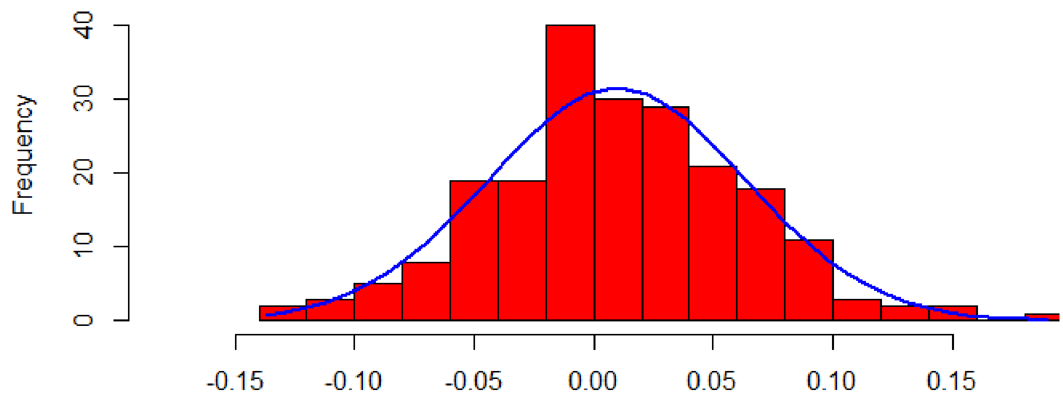
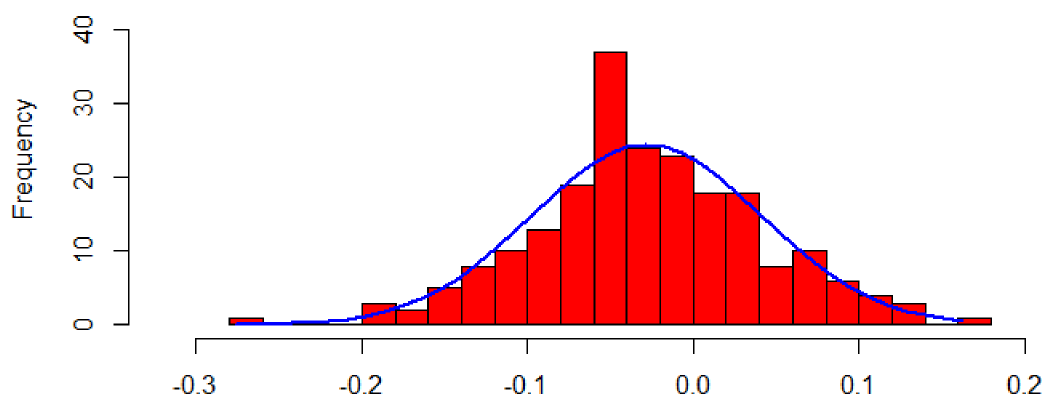


Figure 11: Cross-Correlation Between v(t) and u(t-1)



Figures 10 and 11 display the histograms of the cross-correlations between u_t, v_{t-1} and v_t, u_{t-1} respectively. These cross-correlations are computed from 1330 series of ranks for each stock in the sample. In Figure 10, the cross-correlation coefficients takes values between -0.13 to 0.18. The mode of the cross-correlation is positive which shows that mostly u_t are positively correlated with v_{t-1}. In Figure 11, the cross-correlation coefficients vary between -0.29 to 0.16. The mode of Figure 11 is negative which shows that mostly, v_t and u_{t-1} are correlated negatively.

Figure 12: Time Series of $\hat{\rho}_{11}, \hat{\rho}_{12}$

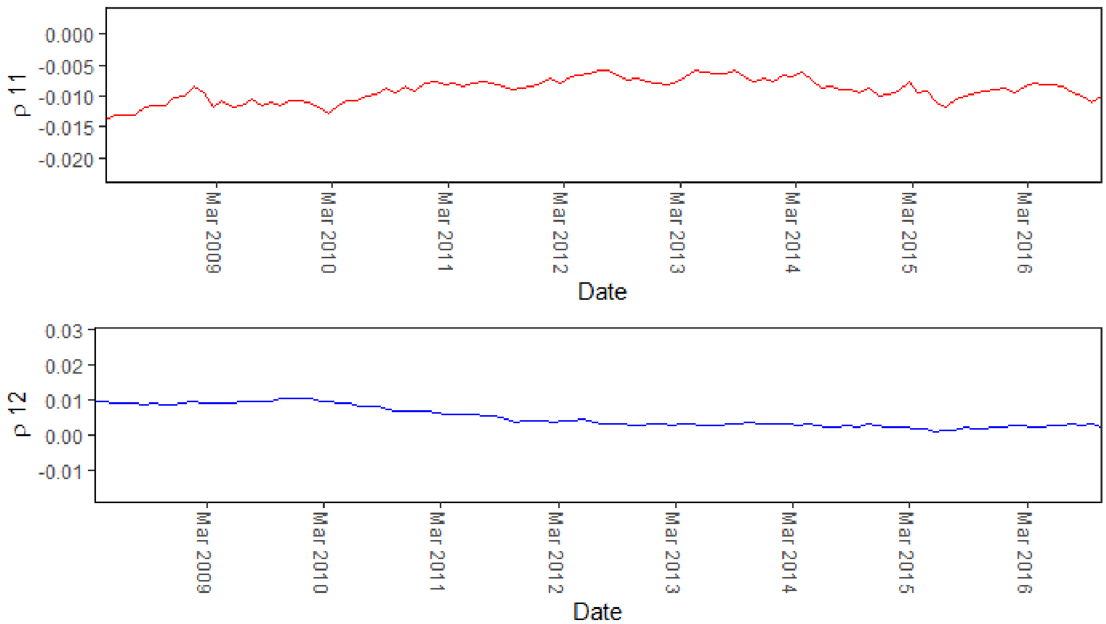
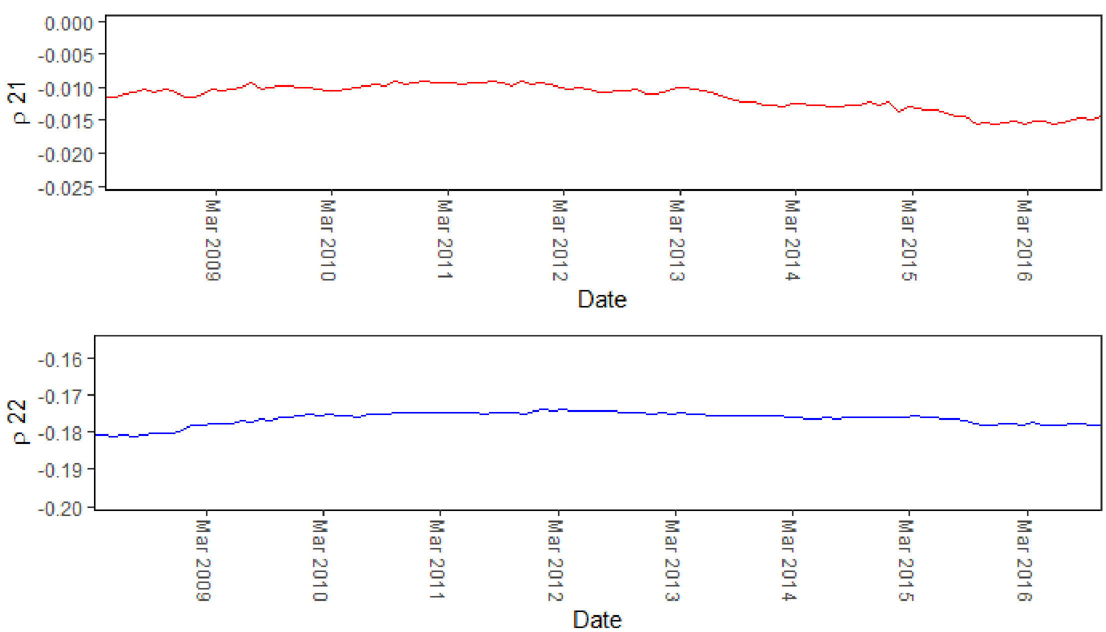


Figure 13: Time Series of $\hat{\rho}_{21}, \hat{\rho}_{22}$



Figures 12 and 13 show the time series of coefficients, which are obtained by re-estimating the model (equation (4.6)) by rolling with the window of 109 months ($\simeq$9 years). Over time, there is little variation and the parameters seem rather stable.

Figure 14: Time Series of $\hat{\eta}$

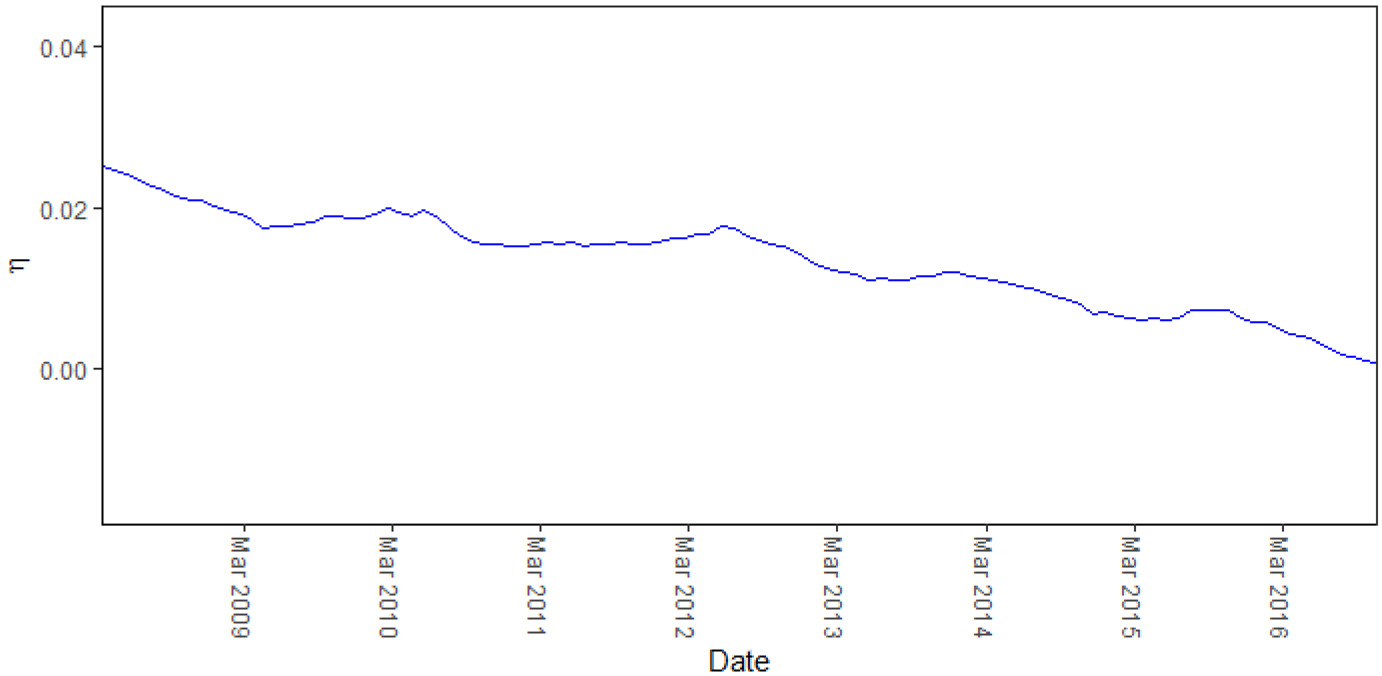


Figure 14 shows the time series of $\hat{\eta}$ (contemporaneous correlation between u_{it} and v_{it}) obtained from the rolling estimation with a window of 109 months. The figure shows a downward trend in $\hat{\eta}$.

Figure 15: Monthly Returns on Positional Momentum Portfolios

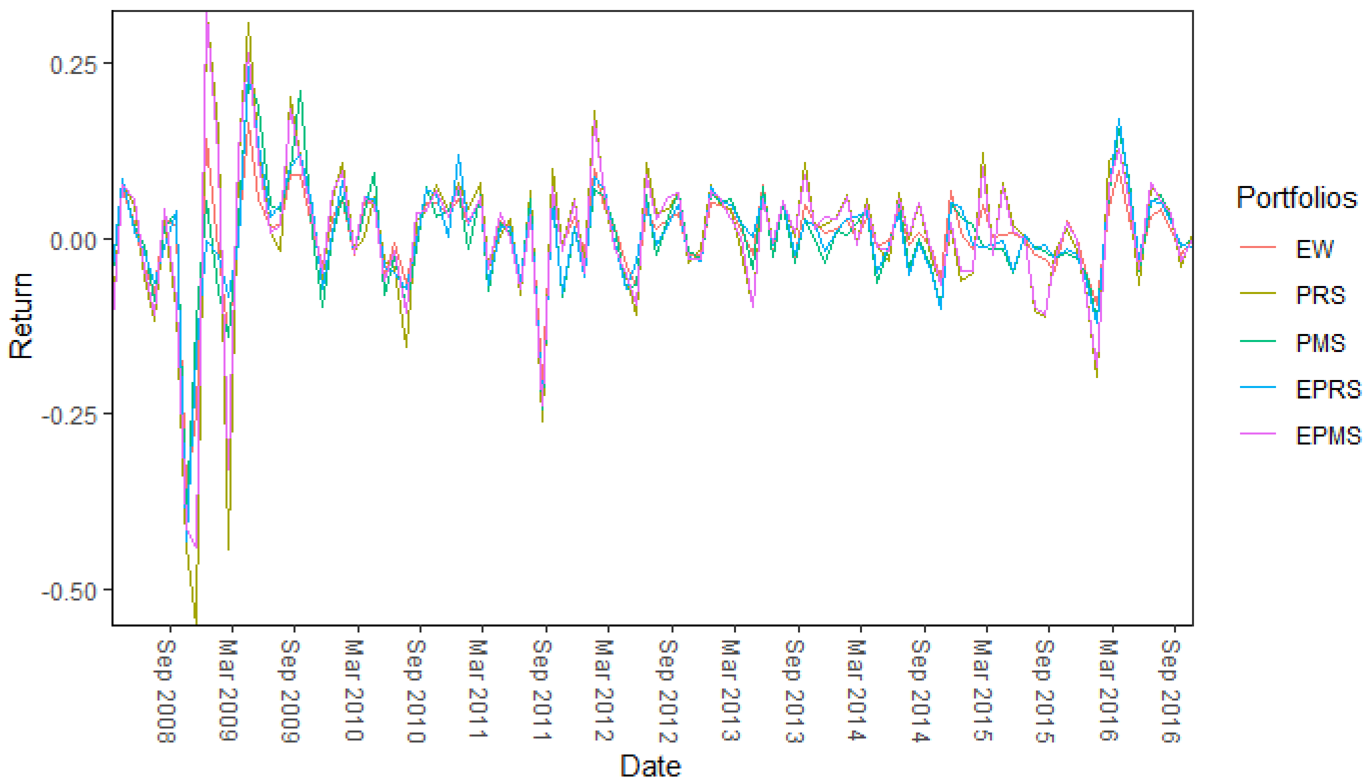


Figure 15 show the time series of monthly portfolio returns generated by the five strategies based on the $VAR(1)$ model from September 2008 to October 2016. The returns on portfolios are color-coded as follows: EW-red, PRS-olive , PMS-green, EPRS-blue, EPMS-purple.

Figure 16: Cumulative Returns on Positional Portfolios From 2008 - VAR(1) Model

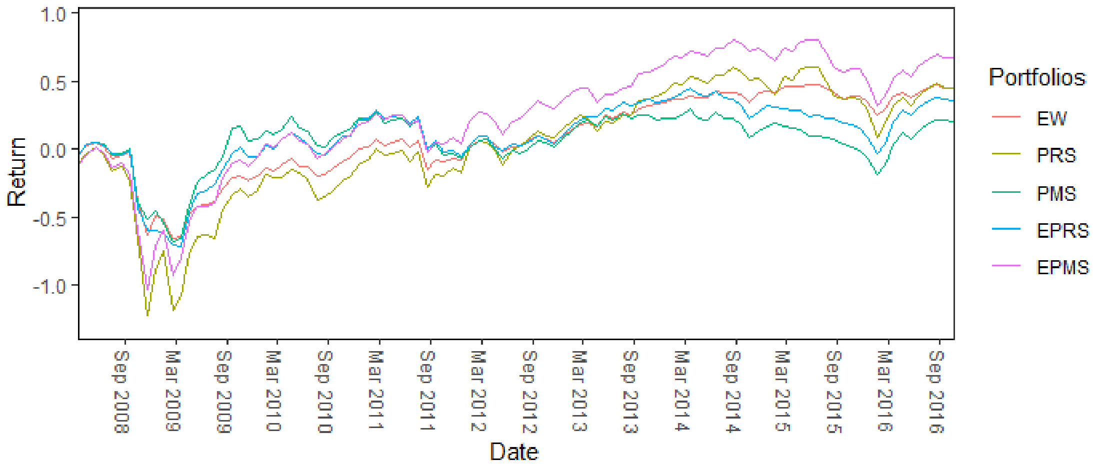
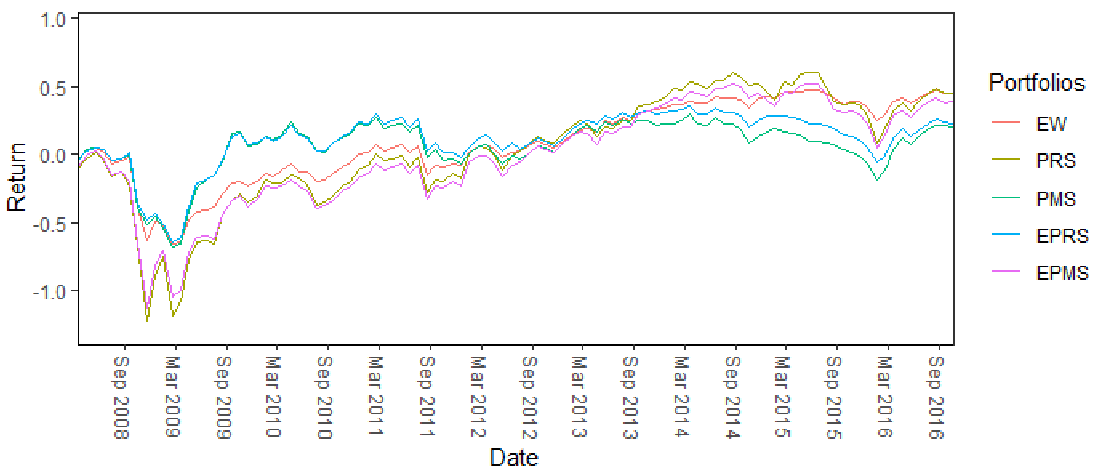


Figure 17: Cumulative Returns on Positional Portfolios From 2008- AR(1) Model



Figures 16 and 17 show the cumulative portfolio returns over time for all positional momentum portfolios with the inception date of January 2008, computed from both the bivariate VAR model and the univariate AR model, respectively. The returns on portfolios are color-coded as follows: EW-red, PRS-olive , PMS-green, EPRS-blue, EPMS-purple. The cumulative return of the VAR-based EPMS outperforms all other strategies after September 2011. During the same period, the PRS provides higher cumulative return than the AR-based strategies as shown in Figure 17.

Figure 18: Cumulative Returns on Positional Liquidity Portfolios From 2008 - VAR(1) Model

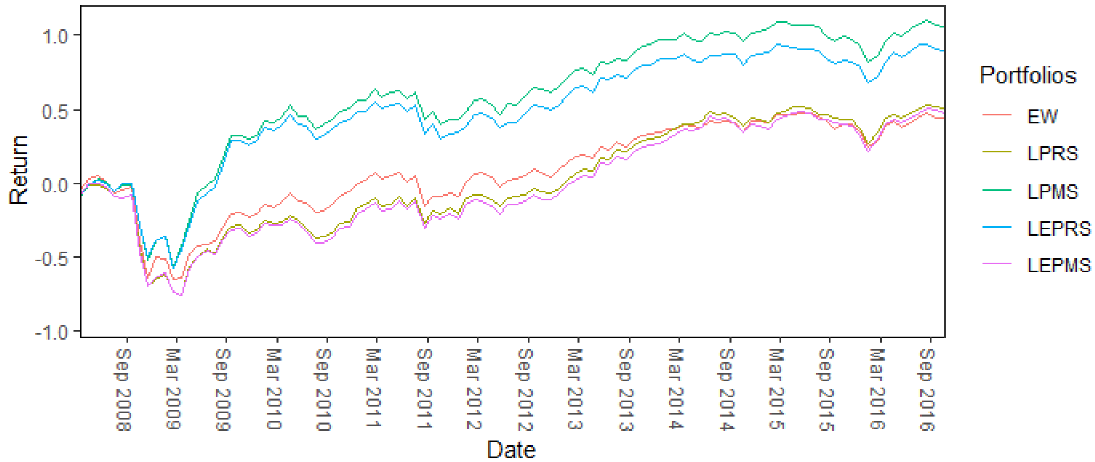
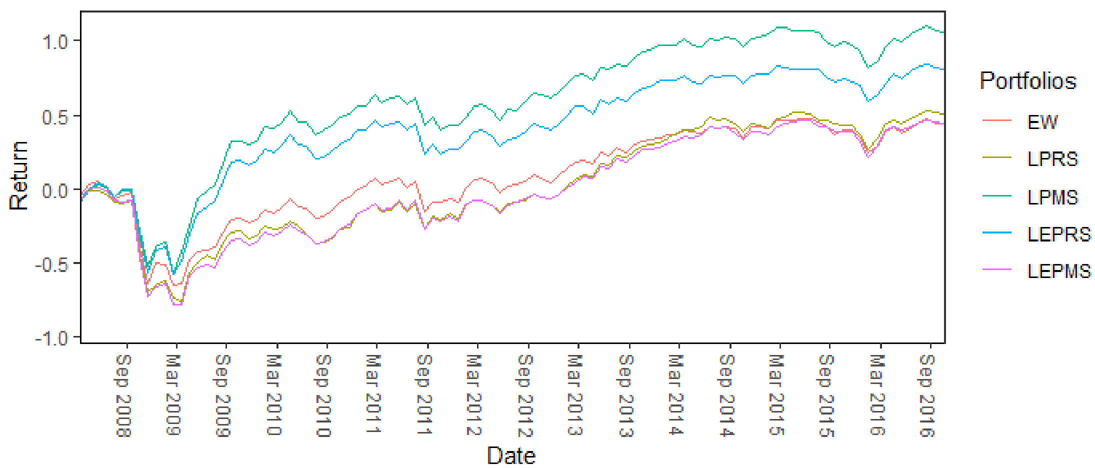


Figure 19: Cumulative Returns on Positional Liquidity Portfolios From 2008 - AR(1) Model



Figures 18 and 19 show the time series of cumulative returns on the positional liquid portfolios held since 2008 and based on the VAR and AR models respectively. The returns on portfolios are color-coded as follow: EW-red, LPRS-olive , LPMS-green, LEPRS-blue, LEPMS-purple. In both Figures the LPMS outperforms other strategies and is followed by the LEPRS.

Appendices

A Diagnostic Tests For Error Terms

A.1 Autocorrelation Test

The Durbin-Watson (DW) statistic is used to test the presence of autocorrelation at lag 1 in the residuals (prediction errors). The null hypothesis is that the errors are serially uncorrelated and the alternative is that they follow a first order autoregressive process. If $\hat{e_t}$ is the residual associated with the observation at time t, then the test statistic is:

$$d = \frac{\sum_{i=2}^{T}(\hat{e}_t - \hat{e}_{t-1})^2}{\sum_{i=1}^{T} \hat{e}_t^2} \tag{A.18}$$

where T is the number of observations. Since d is approximately equal to $2(1-r)$, where r is the sample autocorrelation of the residuals, $d = 2$ indicates no autocorrelation. From model(4.6) it follows that bivariate residual is $\hat{e}_t = \begin{pmatrix} \hat{e}_{1t} \\ \hat{e}_{2t} \end{pmatrix} = \hat{\Sigma}^{-\frac{1}{2}} \left[\begin{pmatrix} u_{it} \\ v_{it} \end{pmatrix} - \hat{R} \begin{pmatrix} u_{i,t-1} \\ v_{i,t-1} \end{pmatrix} \right]$, where $\hat{\Sigma}$ and $\hat{R}$ are the estimation of Σ and R. Then the DW statistics can be computed separately from $\hat{e}_{1t}$ and $\hat{e}_{2t}$. The values of d for the VAR(1) model (equation(4)) are as follows:

$$\begin{aligned} \hat{d}_{e_1} &= 1.99 \\ \hat{d}_{e_2} &= 2.15\ . \end{aligned} \tag{A.19}$$

Since the obtained $d's$ are very close to 2, we conclude that there is no autocorrelation in error terms.

A.2 Normality of Cross-Sectional Errors

We perform the Shapiro Normality test for the cross-sectional errors obtained from VAR(1) model in equation (4.6). I found that the Shapiro test never rejects normality cross-sectionally for $e_{1,it}$ and 42% of times Shapiro rejects normality cross-sectionally for $e_{2,it}$. The following Figures show the Q-Q plot and the distribution function of the cross-sectional residuals in October,2016.

Figure 20: Cross-sectional distribution of $\hat{e}_1$ for Oct,2016

Figure 21: Cross-sectional distribution of $\hat{e}_2$ for Oct,2016

A.3 Normality of Serial Errors for $S\&P500$

The Shapiro test of normality of the distribution of the serial error terms is applied to residuals obtained from the VAR(1) model estimated for $S\&P500$. The test rejects the normality distribution for $\hat{e}_1$ ($p-value = 0.42$), but it did not reject normality distribution of $\hat{e}_2$ since the p-value is 0.02. It means that for $S\&P500$ the distribution of error terms from the return equation of the VAR(1) model is not normal while it is normal for the error terms from the trade volume equation of the VAR(1) model. The following Figures display the Q-Q plot and the historical distribution function of the series of residuals estimated from VAR(1) model for $S\&P500$. Figure 22 shows that the historical error term $\hat{e}_1$ estimated from the VAR(1) model is normal while Figure 23 shows that the historical error term $\hat{e}_2$ estimated from the VAR(1) model is not normal.

Figure 22: Historical Distribution Function for $\hat{e}_1$ for $S\&P500$

Figure 23: Historical Distribution Function for $\hat{e}_2$ for $S\&P500$

B AR(1) Model for Ranks of Return and Trade Volume Change

It is easy to see that the VAR(1) model for return (trade volume) is equivalent to the separate AR(1) models under the condition $\rho_{12} = \rho_{21} = 0$. The following univariate AR(1) models are considered for comparison, with the ranks of return and trade volume dynamics separately:

$$u_{i,t} = \rho_1 u_{i,t-1} + \sqrt{1-\rho_1^2}\epsilon_{1t} \tag{B.20}$$

$$v_{i,t} = \rho_2 v_{i,t-1} + \sqrt{1-\rho_2^2}\epsilon_{2t} \tag{B.21}$$

These equations have been estimated by maximizing the following log likelihood function:

$$\sum_{t=1}^{T}\Big\{-log(2\pi) - \frac{1}{2}log(1-\rho_1^2) - \frac{(\hat{u}_t - \rho_1\hat{u}_{t-1})^2}{2(1-\rho_1^2)}\Big\}. \tag{B.22}$$

$$\sum_{t=1}^{T}\Big\{-log(2\pi) - \frac{1}{2}log(1-\rho_2^2) - \frac{(\hat{v}_t - \rho_1\hat{v}_{t-1})^2}{2(1-\rho_2^2)}\Big\} \tag{B.23}$$

The results of these estimations are provided in Table 9 as follow:

Table 9: Estimated AR(1) Model

Both estimated autoregressive coefficients are strongly significant and negative for both the ranks of return and trade volume changes.

Table 9: Estimated AR(1) Model

$Coefficients$	$Values$
ρ_1	-0.286***
ρ_2	-0.645***

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 9 shows the estimated parameters for univariate AR(1) models for u_{it} and v_{it}. Both estimated parameters are strongly significant. The signs of the parameters are consistent with the signs of $\hat{\rho}_{11}$ and $\hat{\rho}_{22}$.

Figure 20: Cross-Sectional Distribution of $\hat{e}_1$, Oct,2016

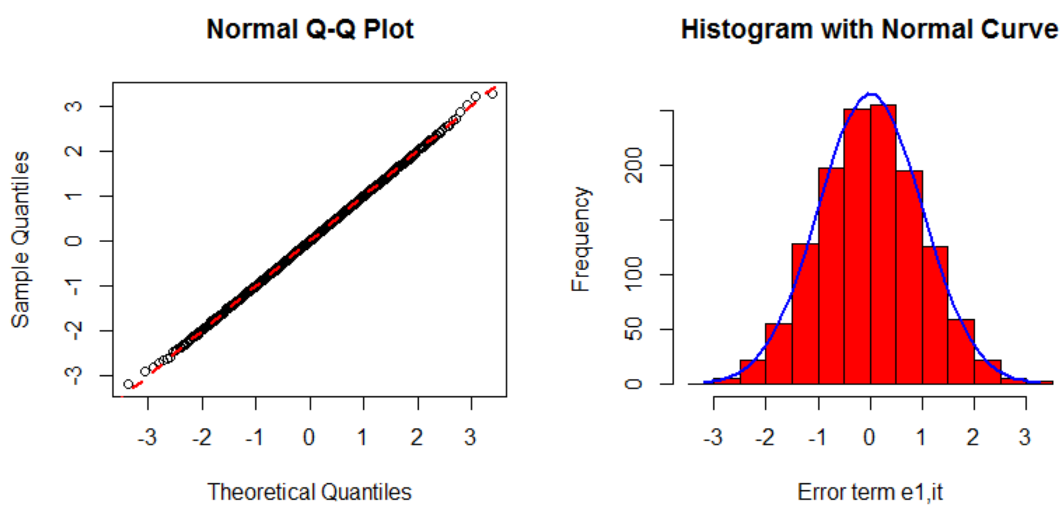


Figure 20 displays the Q-Q plot and cross-sectional histogram of $\hat{e}_1$. The histogram and the Q-Q plot both show that $\hat{e}_1$ is cross-sectionaly Normally distributed.

Figure 21: Cross-Sectional Distribution of $\hat{e}_2$, Oct,2016

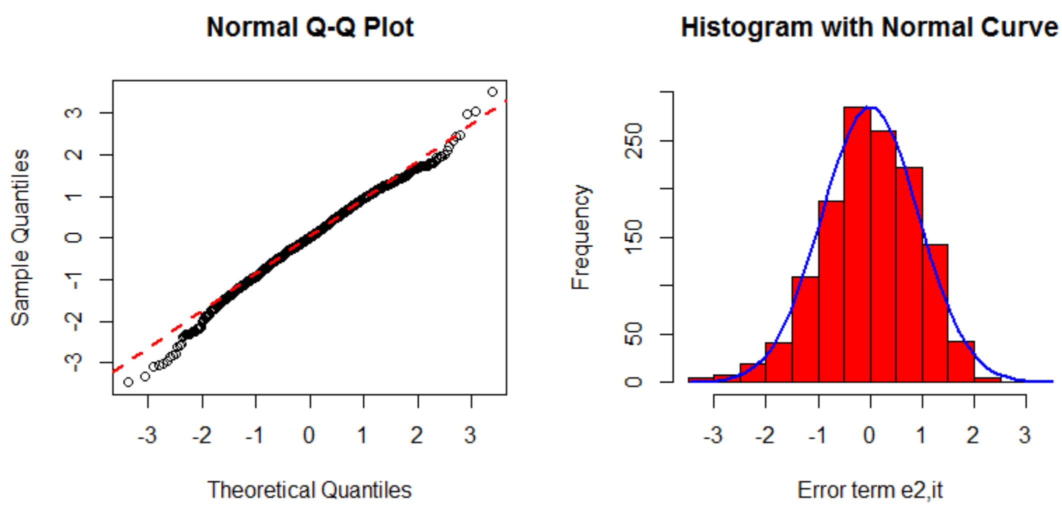


Figure 21 displays the Q-Q plot and cross-sectional histogram of $\hat{e}_2$. The histogram and the Q-Q plot both show that $\hat{e}_2$ is cross-sectionaly Normally distributed.

Figure 22: Historical Distribution Function for $\hat{e}_1$ from $S\&P500$

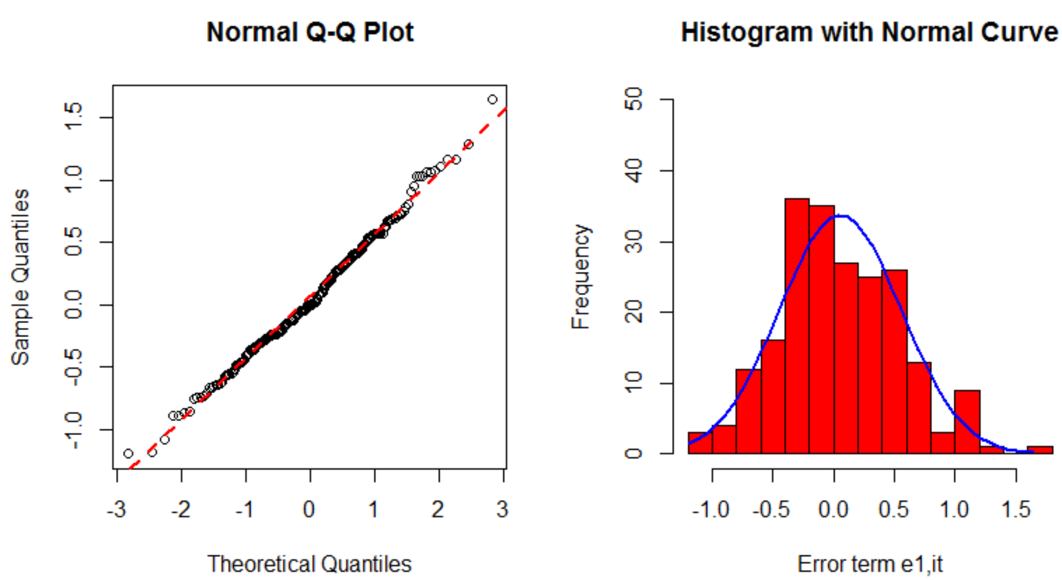


Figure 22 displays the Q-Q plot and cross-sectional histogram of residual $\hat{e}_1$ for S&P.500. The histogram and the Q-Q plot both show that $\hat{e}_1$ is Normally distributed.

Figure 23: Historical Distribution Function for $\hat{e}_2$ from $S\&P500$

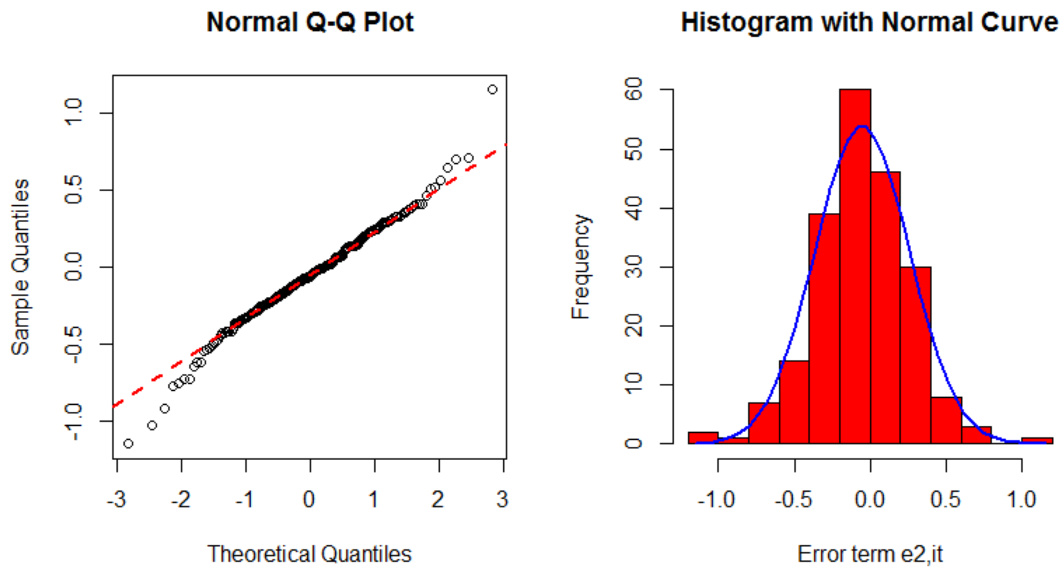


Figure 23 displays the Q-Q plot and cross-sectional histogram of residual $\hat{e}_2$ for S&P.500. The histogram and the Q-Q plot both show that $\hat{e}_2$ is not Normally distributed due to fat tails.