

ADDITIONAL RESULTS on T-Student and $N(0,1)$ Mixtures for Misspecification of Causal and Noncausal Orders in Autoregressive Processes

Christian Gourieroux ^{*} and Joann Jasiak [†]

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Abstract

These tables contain simulation results that clarify the consequences of estimating a (past-dependent) causal AR model from data generated by a stationary (future-dependent) noncausal process. The data are a mixture of a t-student distribution with 3 degrees of freedom and $N(0,1)$.

^{*}University of Toronto and CREST, *e-mail*: gouriero@ensae.fr

[†]York University, Canada, *e-mail*: jasiakj@yorku.ca.

0.1 Estimation results

The degree of freedom of the t-variable is set equal to 3 and the ratio of normals in the sample, denoted by λ , is assumed to be known. We examine the behavior of the autoregressive coefficient estimator $\hat{\rho}(\lambda_0, \rho_0)$ as a function of λ_0 , and the persistence parameter ρ_0 .

We use 1000 replications of noncausal AR(1) trajectories of length $T = 400$ and $T = 100$, which are referred to as the large and the small sample, respectively. The true parameter ρ_0 takes the values 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9. Parameter λ_0 is set equal to the following values: 0.0, 0.01, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.85, 0.9, 0.95 and 1.

Tables 1.0 to 1.9 display the behavior of the PML estimators of $\hat{\rho}(\lambda_0, \rho_0)$, called the Mix PMLE and t-student PMLE and given in columns 1 and 3, respectively, and compare them to the OLS-based first-order autocorrelation estimator, denoted by OLS and given in column 2.

For each estimator, we report the median and the 5th and 95th percentiles of its sampling distribution.

The OLS estimator leads to a misleading conclusion that the model is causal while it is not.

The PML estimators underestimate the autoregressive coefficient when it is below 0.5 and overestimate it otherwise.

Table 1.0: Estimates of $\rho = 0.0$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			OLS			t-stud PMLE		
0.0000	0.0037	-0.0560	0.0637	0.0010	-0.0765	0.0780	0.0037	-0.0560	0.0637
0.0100	0.0035	-0.0562	0.0663	0.0016	-0.0757	0.0789	0.0035	-0.0562	0.0663
0.1000	0.0028	-0.0581	0.0695	-0.0001	-0.0764	0.0828	0.0029	-0.0579	0.0696
0.2000	0.0042	-0.0640	0.0688	0.0023	-0.0817	0.0773	0.0044	-0.0635	0.0689
0.3000	0.0015	-0.0648	0.0666	-0.0007	-0.0817	0.0778	0.0018	-0.0647	0.0664
0.4000	0.0023	-0.0685	0.0706	0.0002	-0.0796	0.0791	0.0022	-0.0688	0.0706
0.5000	0.0010	-0.0703	0.0684	0.0006	-0.0791	0.0786	0.0012	-0.0695	0.0692
0.6000	0.0047	-0.0713	0.0750	0.0027	-0.0778	0.0807	0.0051	-0.0705	0.0762
0.7000	0.0025	-0.0732	0.0811	0.0009	-0.0808	0.0790	0.0028	-0.0746	0.0785
0.8000	0.0014	-0.0771	0.0783	-0.0007	-0.0784	0.0849	0.0020	-0.0778	0.0789
0.8500	0.0017	-0.0749	0.0760	-0.0025	-0.0847	0.0736	0.0023	-0.0758	0.0793
0.9000	-0.0001	-0.0748	0.0825	-0.0023	-0.0826	0.0755	-0.0006	-0.0768	0.0845
0.9500	0.0030	-0.0822	0.0815	0.0006	-0.0883	0.0825	0.0029	-0.0850	0.0829
1.0000	0.0020	-0.0781	0.0840	-0.0004	-0.0807	0.0801	0.0004	-0.0791	0.0866

Table 1.1: Estimates of $\rho = 0.1$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
T=400									
	Mix PMLE			OLS			t-stud PMLE		
0.0000	0.0550	-0.0032	0.1207	0.0979	0.0209	0.1797	0.0550	-0.0032	0.1207
0.0100	0.0557	-0.0029	0.1215	0.0996	0.0198	0.1762	0.0557	-0.0028	0.1215
0.1000	0.0571	-0.0053	0.1234	0.1006	0.0154	0.1745	0.0570	-0.0051	0.1235
0.2000	0.0594	-0.0074	0.1303	0.0986	0.0221	0.1807	0.0597	-0.0063	0.1304
0.3000	0.0584	-0.0059	0.1326	0.0952	0.0215	0.1784	0.0591	-0.0058	0.1333
0.4000	0.0644	-0.0073	0.1319	0.0947	0.0175	0.1778	0.0653	-0.0062	0.1322
0.5000	0.0664	-0.0068	0.1418	0.0997	0.0180	0.1780	0.0679	-0.0060	0.1431
0.6000	0.0714	-0.0012	0.1504	0.0997	0.0166	0.1875	0.0734	-0.0002	0.1507
0.7000	0.0728	0.0002	0.1514	0.0971	0.0186	0.1737	0.0733	0.0022	0.1545
0.8000	0.0816	0.0015	0.1618	0.0973	0.0175	0.1779	0.0815	0.0002	0.1625
0.8500	0.0829	0.0076	0.1671	0.0952	0.0136	0.1811	0.0838	0.0036	0.1703
0.9000	0.0905	0.0124	0.1684	0.0988	0.0177	0.1721	0.0919	0.0098	0.1732
0.9500	0.0942	0.0104	0.1759	0.0983	0.0120	0.1762	0.0943	0.0101	0.1758
1.0000	0.1028	0.0133	0.1819	0.0990	0.0108	0.1794	0.1016	0.0072	0.1849

Table 1.2: Estimates of $\rho = 0.2$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			OLS			t-stud PMLE		
0.0000	0.1194	0.0452	0.1962	0.1937	0.1169	0.2782	0.1194	0.0452	0.1962
0.0100	0.1197	0.0474	0.1980	0.1925	0.1157	0.2765	0.1198	0.0474	0.1981
0.1000	0.1246	0.0446	0.2036	0.1970	0.1164	0.2764	0.1253	0.0456	0.2037
0.2000	0.1263	0.0542	0.2062	0.1980	0.1180	0.2742	0.1269	0.0554	0.2068
0.3000	0.1279	0.0513	0.2145	0.1931	0.1220	0.2754	0.1292	0.0530	0.2158
0.4000	0.1383	0.0525	0.2168	0.1948	0.1168	0.2687	0.1401	0.0545	0.2183
0.5000	0.1405	0.0601	0.2277	0.1942	0.1149	0.2735	0.1427	0.0600	0.2294
0.6000	0.1492	0.0667	0.2359	0.1980	0.1180	0.2736	0.1513	0.0689	0.2377
0.7000	0.1621	0.0761	0.2464	0.1985	0.1171	0.2717	0.1637	0.0788	0.2482
0.8000	0.1707	0.0882	0.2524	0.1964	0.1132	0.2747	0.1731	0.0889	0.2550
0.8500	0.1754	0.0931	0.2560	0.1946	0.1165	0.2726	0.1761	0.0938	0.2602
0.9000	0.1835	0.0940	0.2676	0.1981	0.1158	0.2757	0.1839	0.0951	0.2715
0.9500	0.1900	0.1058	0.2773	0.1931	0.1166	0.2781	0.1913	0.1052	0.2776
1.0000	0.1992	0.1202	0.2778	0.1949	0.1165	0.2750	0.2000	0.1163	0.2827

Table 1.3: Estimates of $\rho = 0.3$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
	T=400								
	Mix PMLE			OLS			t-stud PMLE		
0.0000	0.2089	0.1245	0.2938	0.2969	0.2140	0.3693	0.2090	0.1245	0.2938
0.0100	0.2086	0.1240	0.2935	0.2964	0.2175	0.3670	0.2087	0.1239	0.2936
0.1000	0.2127	0.1233	0.2924	0.2975	0.2213	0.3664	0.2132	0.1238	0.2930
0.2000	0.2158	0.1257	0.3019	0.2949	0.2158	0.3659	0.2166	0.1270	0.3032
0.3000	0.2230	0.1294	0.3102	0.2964	0.2172	0.3698	0.2246	0.1309	0.3118
0.4000	0.2299	0.1376	0.3106	0.2962	0.2138	0.3664	0.2314	0.1401	0.3130
0.5000	0.2376	0.1410	0.3306	0.2968	0.2165	0.3685	0.2398	0.1462	0.3312
0.6000	0.2430	0.1525	0.3288	0.2953	0.2128	0.3672	0.2446	0.1549	0.3315
0.7000	0.2570	0.1663	0.3474	0.2973	0.2180	0.3716	0.2598	0.1665	0.3467
0.8000	0.2684	0.1797	0.3525	0.2935	0.2121	0.3681	0.2702	0.1833	0.3531
0.8500	0.2728	0.1855	0.3571	0.2946	0.2181	0.3698	0.2756	0.1853	0.3563
0.9000	0.2812	0.1978	0.3594	0.2947	0.2162	0.3667	0.2832	0.1985	0.3610
0.9500	0.2890	0.2055	0.3708	0.2950	0.2180	0.3721	0.2905	0.2039	0.3728
1.0000	0.2974	0.2218	0.3714	0.2935	0.2173	0.3672	0.2976	0.2168	0.3747

Table 1.4: Estimates of $\rho = 0.4$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
T=400									
	Mix PMLE			OLS			t-stud PMLE		
0.0000	0.3182	0.2191	0.4195	0.3925	0.3153	0.4645	0.3182	0.2191	0.4195
0.0100	0.3193	0.2237	0.4207	0.3919	0.3147	0.4643	0.3192	0.2237	0.4207
0.1000	0.3235	0.2280	0.4221	0.3953	0.3155	0.4664	0.3243	0.2288	0.4219
0.2000	0.3266	0.2298	0.4205	0.3936	0.3187	0.4638	0.3269	0.2324	0.4206
0.3000	0.3350	0.2382	0.4319	0.3934	0.3151	0.4675	0.3365	0.2415	0.4337
0.4000	0.3404	0.2411	0.4336	0.3952	0.3155	0.4634	0.3416	0.2434	0.4329
0.5000	0.3474	0.2509	0.4388	0.3948	0.3165	0.4657	0.3490	0.2564	0.4404
0.6000	0.3580	0.2638	0.4399	0.3951	0.3098	0.4631	0.3590	0.2662	0.4425
0.7000	0.3630	0.2698	0.4492	0.3947	0.3146	0.4634	0.3660	0.2737	0.4464
0.8000	0.3740	0.2874	0.4565	0.3951	0.3191	0.4618	0.3758	0.2893	0.4547
0.8500	0.3800	0.2957	0.4635	0.3935	0.3159	0.4676	0.3802	0.2948	0.4643
0.9000	0.3884	0.2959	0.4661	0.3950	0.3207	0.4634	0.3881	0.2980	0.4647
0.9500	0.3926	0.3119	0.4722	0.3934	0.3152	0.4701	0.3940	0.3096	0.4719
1.0000	0.3976	0.3224	0.4734	0.3931	0.3170	0.4692	0.3996	0.3182	0.4762

Table 1.5: Estimates of $\rho = 0.5$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
T=400									
	Mix PMLE			OLS			t-stud PMLE		
0.0000	0.4658	0.3569	0.5600	0.4934	0.4150	0.5610	0.4658	0.3569	0.5600
0.0100	0.4666	0.3583	0.5576	0.4927	0.4149	0.5613	0.4667	0.3582	0.5575
0.1000	0.4672	0.3624	0.5591	0.4928	0.4213	0.5575	0.4678	0.3632	0.5580
0.2000	0.4705	0.3683	0.5579	0.4929	0.4171	0.5585	0.4702	0.3708	0.5577
0.3000	0.4707	0.3731	0.5648	0.4911	0.4205	0.5622	0.4715	0.3764	0.5639
0.4000	0.4704	0.3800	0.5631	0.4913	0.4223	0.5572	0.4705	0.3838	0.5618
0.5000	0.4767	0.3835	0.5658	0.4924	0.4182	0.5581	0.4779	0.3857	0.5643
0.6000	0.4784	0.3889	0.5712	0.4932	0.4207	0.5544	0.4793	0.3948	0.5683
0.7000	0.4840	0.3942	0.5600	0.4909	0.4127	0.5562	0.4856	0.3961	0.5596
0.8000	0.4875	0.4055	0.5656	0.4917	0.4220	0.5603	0.4877	0.4080	0.5653
0.8500	0.4901	0.4102	0.5639	0.4925	0.4225	0.5584	0.4901	0.4128	0.5610
0.9000	0.4916	0.4141	0.5679	0.4932	0.4259	0.5589	0.4919	0.4127	0.5686
0.9500	0.4936	0.4171	0.5626	0.4937	0.4218	0.5569	0.4944	0.4167	0.5667
1.0000	0.4980	0.4235	0.5643	0.4924	0.4185	0.5579	0.4978	0.4143	0.5652

Table 1.6: Estimates of $\rho = 0.6$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
T=400									
	Mix PMLE			OLS			t-stud PMLE		
0.0000	0.6347	0.5313	0.7317	0.5938	0.5220	0.6539	0.6347	0.5313	0.7317
0.0100	0.6327	0.5306	0.7313	0.5934	0.5236	0.6546	0.6327	0.5306	0.7312
0.1000	0.6314	0.5275	0.7248	0.5933	0.5247	0.6560	0.6310	0.5275	0.7239
0.2000	0.6276	0.5255	0.7142	0.5913	0.5223	0.6544	0.6267	0.5256	0.7131
0.3000	0.6265	0.5286	0.7141	0.5949	0.5215	0.6555	0.6246	0.5302	0.7116
0.4000	0.6227	0.5326	0.7103	0.5925	0.5237	0.6521	0.6209	0.5331	0.7071
0.5000	0.6180	0.5305	0.7109	0.5910	0.5210	0.6513	0.6173	0.5310	0.7077
0.6000	0.6134	0.5208	0.6927	0.5904	0.5197	0.6526	0.6119	0.5228	0.6888
0.7000	0.6108	0.5293	0.6935	0.5917	0.5213	0.6562	0.6113	0.5283	0.6900
0.8000	0.6039	0.5211	0.6796	0.5898	0.5185	0.6527	0.6039	0.5229	0.6785
0.8500	0.6060	0.5252	0.6774	0.5933	0.5240	0.6542	0.6050	0.5242	0.6737
0.9000	0.6024	0.5271	0.6713	0.5918	0.5157	0.6535	0.6012	0.5270	0.6726
0.9500	0.6008	0.5291	0.6643	0.5918	0.5234	0.6531	0.5995	0.5289	0.6646
1.0000	0.5985	0.5255	0.6572	0.5930	0.5221	0.6527	0.5964	0.5247	0.6612

Table 1.7: Estimates of $\rho = 0.7$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
T=400									
	Mix PMLE			OLS			t-stud PMLE		
0.0000	0.7897	0.6946	0.8904	0.6920	0.6263	0.7441	0.7897	0.6946	0.8904
0.0100	0.7893	0.6946	0.8923	0.6920	0.6273	0.7440	0.7892	0.6945	0.8921
0.1000	0.7871	0.6979	0.8808	0.6942	0.6273	0.7474	0.7861	0.6974	0.8799
0.2000	0.7802	0.6882	0.8778	0.6918	0.6269	0.7468	0.7786	0.6872	0.8751
0.3000	0.7725	0.6833	0.8673	0.6904	0.6268	0.7494	0.7701	0.6815	0.8648
0.4000	0.7676	0.6724	0.8511	0.6923	0.6255	0.7466	0.7647	0.6719	0.8478
0.5000	0.7580	0.6785	0.8399	0.6903	0.6280	0.7455	0.7543	0.6770	0.8366
0.6000	0.7464	0.6672	0.8321	0.6906	0.6258	0.7491	0.7436	0.6665	0.8301
0.7000	0.7369	0.6593	0.8096	0.6930	0.6276	0.7457	0.7339	0.6588	0.8083
0.8000	0.7243	0.6479	0.7911	0.6929	0.6268	0.7411	0.7234	0.6456	0.7901
0.8500	0.7204	0.6397	0.7862	0.6945	0.6247	0.7481	0.7200	0.6384	0.7844
0.9000	0.7121	0.6428	0.7771	0.6932	0.6265	0.7470	0.7108	0.6413	0.7779
0.9500	0.7068	0.6397	0.7642	0.6943	0.6298	0.7491	0.7051	0.6384	0.7655
1.0000	0.6996	0.6326	0.7540	0.6933	0.6254	0.7479	0.6990	0.6284	0.7556

Table 1.8: Estimates of $\rho = 0.8$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
T=400									
	Mix PMLE			OLS			t-stud PMLE		
0.0000	0.9116	0.8353	0.9941	0.7911	0.7376	0.8351	0.9116	0.8353	0.9940
0.0100	0.9100	0.8365	0.9940	0.7918	0.7348	0.8349	0.9098	0.8364	0.9939
0.1000	0.9053	0.8314	0.9908	0.7904	0.7374	0.8359	0.9049	0.8310	0.9903
0.2000	0.9033	0.8243	0.9863	0.7902	0.7396	0.8364	0.9012	0.8234	0.9841
0.3000	0.8934	0.8212	0.9763	0.7916	0.7385	0.8350	0.8908	0.8190	0.9753
0.4000	0.8870	0.8059	0.9645	0.7927	0.7354	0.8366	0.8842	0.8030	0.9614
0.5000	0.8775	0.8025	0.9549	0.7907	0.7338	0.8343	0.8733	0.7969	0.9522
0.6000	0.8621	0.7928	0.9415	0.7906	0.7348	0.8367	0.8585	0.7906	0.9408
0.7000	0.8514	0.7816	0.9263	0.7913	0.7326	0.8361	0.8480	0.7807	0.9238
0.8000	0.8354	0.7692	0.9038	0.7929	0.7348	0.8388	0.8330	0.7677	0.9019
0.8500	0.8269	0.7610	0.8954	0.7917	0.7352	0.8357	0.8251	0.7605	0.8944
0.9000	0.8190	0.7539	0.8767	0.7918	0.7335	0.8387	0.8174	0.7533	0.8771
0.9500	0.8079	0.7507	0.8628	0.7904	0.7352	0.8384	0.8080	0.7496	0.8614
1.0000	0.7985	0.7409	0.8456	0.7920	0.7330	0.8394	0.7988	0.7402	0.8470

Table 1.9: Estimates of $\rho = 0.9$

lambda	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)	med	q(0.05)	q(0.95)
T=400									
	Mix PMLE			OLS			t-stud PMLE		
0.0000	0.9757	0.9293	1.0264	0.8921	0.8466	0.9229	0.9757	0.9293	1.0264
0.0100	0.9760	0.9298	1.0268	0.8921	0.8461	0.9224	0.9759	0.9298	1.0267
0.1000	0.9729	0.9228	1.0252	0.8921	0.8467	0.9216	0.9723	0.9222	1.0245
0.2000	0.9696	0.9204	1.0243	0.8908	0.8481	0.9234	0.9684	0.9193	1.0227
0.3000	0.9653	0.9157	1.0207	0.8904	0.8461	0.9224	0.9638	0.9143	1.0193
0.4000	0.9618	0.9132	1.0098	0.8928	0.8466	0.9220	0.9599	0.9119	1.0073
0.5000	0.9556	0.9048	1.0062	0.8923	0.8499	0.9212	0.9535	0.9023	1.0047
0.6000	0.9471	0.9005	1.0025	0.8920	0.8493	0.9231	0.9450	0.8986	0.9991
0.7000	0.9372	0.8838	0.9894	0.8903	0.8451	0.9219	0.9350	0.8840	0.9853
0.8000	0.9275	0.8778	0.9771	0.8920	0.8471	0.9223	0.9259	0.8757	0.9728
0.8500	0.9204	0.8714	0.9674	0.8913	0.8482	0.9226	0.9195	0.8709	0.9674
0.9000	0.9132	0.8669	0.9572	0.8910	0.8465	0.9225	0.9121	0.8664	0.9548
0.9500	0.9056	0.8594	0.9473	0.8907	0.8472	0.9236	0.9051	0.8569	0.9482
1.0000	0.8981	0.8543	0.9280	0.8906	0.8461	0.9227	0.8981	0.8512	0.9289