Dynamic Deconvolution and Identification of Independent Autoregressive Sources
Online Appendix

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Full Rank of the System

Let us consider the (K,K) matrix:

$$C = \begin{pmatrix} 1 & 1 & \dots & 1 \\ \rho_1 & \rho_2 & \dots & \rho_K \\ \vdots & \vdots & \vdots & \vdots \\ \rho_1^{K-1} & \rho_2^{K-1} & \dots & \rho_K^{K-1} \end{pmatrix}.$$

Lemma: The matrix C is invertible if and only if $\rho_l \neq \rho_k$ for any $k \neq l$.

Proof: The determinant of C is a polynomial in $\rho_1, ..., \rho_K$ of degree $1 + 2 + \ldots + (K-1) = K(K-1)/2$. Moreover if $\rho_k = \rho_l$, two columns of matrix C are equal, and then $det\ C = 0$. It follows that the polynomial determinant of C is divisible by $\rho_k - \rho_l$ for any pair $k \neq l$. Since there are $K(K-1)/2$ such pairs, $det\ C$ is equal to $\prod_{k<l}(\rho_k - \rho_l)$ up to a multiplicative scalar, by the fundamental theorem of algebra. Therefore, it is different from 0, if and only if $\rho_k \neq \rho_l$ for any $k \neq l$.

QED